Name _____ Class _____ Date _____

Pre-Calculus: Completing the Square Review

Completing a perfect square trinomial allows you to factor the completed trinomial as the square of a binomial.

Start with the expression $x^2 + bx$. Add $\left(\frac{b}{2}\right)^2$. Now the expression is $x^2 + bx + \left(\frac{b}{2}\right)^2$,

which can be factored into the square of a binomial: $x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$.

To complete the square for an expression $ax^2 + abx$, first factor out a. Then find the value that completes the square for the factored expression.

Problem

What value completes the square for $-2x^2 + 10x$?

Think	Write
Write the expression in the form $a(x^2 + bx)$.	$-2x^2 + 10x = -2(x^2 - 5x)$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-5}{2} = -\frac{5}{2}$
Add $\left(\frac{b}{2}\right)^2$ to the inner expression to complete the square.	$-2\left[x^2 - 5x + \left(-\frac{5}{2}\right)^2\right] = -2\left(x^2 - 5x + \frac{25}{4}\right)$
Factor the perfect square trinomial.	$-2\left(x - \frac{5}{2}\right)^2$
Find the value that completes the square.	$-2\left(\frac{25}{4}\right) = -\frac{25}{2}$

Exercises

What value completes the square for each expression?

1. $x^2 + 2x$ **2.** $x^2 - 24x$ **3.** $x^2 + 12x$

4. $x^2 - 20x$ **5.** $x^2 + 5x$ **6.** $x^2 - 9x$

7. $2x^2 - 24x$ **8.** $3x^2 + 12x$ **9.** $-x^2 + 6x$

10. $5x^2 + 80x$ **11.** $-7x^2 + 14x$ **12.** $-3x^2 - 15x$

You can easily graph a quadratic function if you first write it in vertex form. Complete the square to change a function in standard form into a function in vertex form.

Problem

What is $y = x^2 - 6x + 14$ in vertex form?

Think	Write
Write an expression using the terms that contain x.	$x^2 - 6x$
Find $\frac{b}{2}$.	$\frac{b}{2} = \frac{-6}{2} = -3$
Add $\left(\frac{b}{2}\right)^2$ to the expression to complete the square.	$x^2 - 6x + (-3)^2 = x^2 - 6x + 9$
Subtract 9 from the expression so that the equation is unchanged.	$y = x^2 - 6x + 9 + 14 - 9$
Factor the perfect square trinomial.	$y = (x - 3)^2 + 14 - 9$
Add the remaining constant terms.	$y = (x - 3)^2 + 5$

Exercises

Rewrite each equation in vertex form.

13. $y = x^2 + 4x + 3$

14. $y = x^2 - 6x + 13$

15. $y = 2x^2 + 4x - 10$

16. $y = x^2 - 2x - 3$

17. $y = x^2 + 8x + 13$

18. $y = -x^2 - 6x - 4$

19. $y = -x^2 + 10x - 18$

20. $y = x^2 + 2x - 8$

21. $y = 2x^2 + 4x - 3$

22. $y = 3x^2 - 12x + 8$