Finding Slope from 2 Points

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Special Cases:

$\frac{0}{\#}$ → slope = 0 $\frac{\#}{0}$ → slope is undefined

Ex: Find the slope of the line that passes through the points (-9, -3) and (7, -7)

$$m = \frac{-7 - (-3)}{7 - (-9)} = \frac{-4}{16} = \boxed{-\frac{1}{4}}$$

Slope-Intercept Form

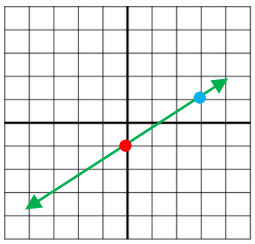
$y = mx + b$

m = slope & b = y-intercept

Graphing from Slope-Intercept Form:

1. Make a point at the y-intercept.
2. Use the slope $\left(\frac{\text{rise}}{\text{run}}\right)$ to make more points.
3. Connect the points to form a line.

Ex: Graph $y = \frac{2}{3}x - 1$



y-intercept is -1
slope = $\frac{2}{3}$, (so from the y-intercept go up 2 & right 3)

Standard Form

$Ax + By = C$

A, B, & C are integers & A is not negative

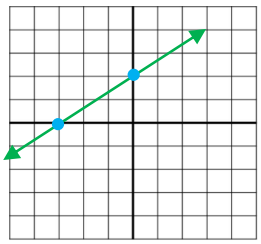
Graphing Using Intercepts:

1. Find the x-intercept by substituting 0 for y.
2. Find the y-intercept by substituting 0 for x.
3. Make a point at each intercept and then connect the points to form a line.

Ex: Graph $2x - 3y = -6$

x-intercept: $2x - 3(0) = -6$
$2x = -6$ → $x = -3$
(-3, 0)

y-intercept: $2(0) - 3y = -6$
$-3y = -6$ → $y = 2$
(0, 2)



Point-Slope Form

$y - y_1 = m(x - x_1)$

m = slope & (x_1, y_1) is a point on the graph

Converting Point-Slope Form to Slope-Intercept Form:

1. Distribute m.
2. Move y_1 to the other side of the equation.

Converting Slope-Intercept Form to Standard Form:

1. Bring the x term to the left.
2. If there are fractions in the equation, multiply everything through by the least common denominator.
3. If A is negative, multiply everything through by -1.

Ex: Write the equation of the line passing through the points (-1, 2) and (3, 4) in point-slope form. Then convert it to slope-intercept and standard form.

$$m = \frac{4 - 2}{3 - (-1)} = \frac{2}{4} = \frac{1}{2}$$

Point-Slope Form: $\boxed{y - 2 = \frac{1}{2}(x + 1)}$

Convert to Slope-Intercept Form:

→ $y - 2 = \frac{1}{2}x + \frac{1}{2}$ → $\boxed{y = \frac{1}{2}x + \frac{5}{2}}$

Convert to Standard Form:

→ $-2\left(-\frac{1}{2}x + y = \frac{5}{2}\right)$ → $\boxed{x - 2y = -5}$

Parallel & Perpendicular Lines

Parallel Lines have the *same slope* but different y-intercepts.

Perpendicular Lines have *opposite reciprocal slopes*.

Writing Equations of Parallel Lines:

1. Find the slope of the original line by first converting it to slope-intercept form if it is in Standard Form. The slope of the line parallel will have that same slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Writing Equations of Perpendicular Lines:

1. Find the slope of the original line. The slope of the line perpendicular will have the opposite (negative) reciprocal slope.
2. Use the given point along with the slope you just found to write the equation of the line in point-slope form.
3. Convert the point-slope form equation to slope-intercept form.

Ex: Write the equation of the line that is parallel to the line $y = 3x - 5$ and passes through the point $(-2, 4)$.

$y = 3x - 5$

$m = 3$, so slope of parallel line is 3, too

→ $y - 4 = 3(x + 2)$

→ $y - 4 = 3x + 6$

→ $\boxed{y = 3x + 10}$

Ex: Write the equation of the line that is perpendicular to the line $x - 3y = -6$ and passes through the point $(-1, 1)$.

$x - 3y = -6$ → $-3y = -x - 6$

→ $y = \frac{1}{3}x + 2$

$m = \frac{1}{3}$, so slope of perpendicular line is -3

→ $y - 1 = -3(x + 1)$

→ $y - 1 = -3x - 3$

→ $\boxed{y = -3x - 2}$

Linear Inequalities

1. Convert the linear inequality in slope-intercept form. Be sure the y is on the left and remember to flip the inequality sign if you multiply or divide by a negative.
2. Graph the line as if it is an equation, except use a dotted line if the inequality sign is < or >. If the sign is ≤ or ≥, use a regular solid line.
3. Shade above the line for a "greater than" inequality (> or ≥). Shade below the line for a "less than" inequality (< or ≤). (For vertical lines, shade to the right for greater than and to the left for less than).

Ex: $-3x - 2y > 8$

$-3x - 2y > 8$

$+3x \quad +3x$

$-2y > 3x + 8$

$\frac{-2y}{-2} > \frac{3x}{-2} + \frac{8}{-2}$

$y < -\frac{3}{2}x - 4$