

Properties of Exponential Functions

$$y = ab^{(x-h)} + k$$

b is part of the parent function

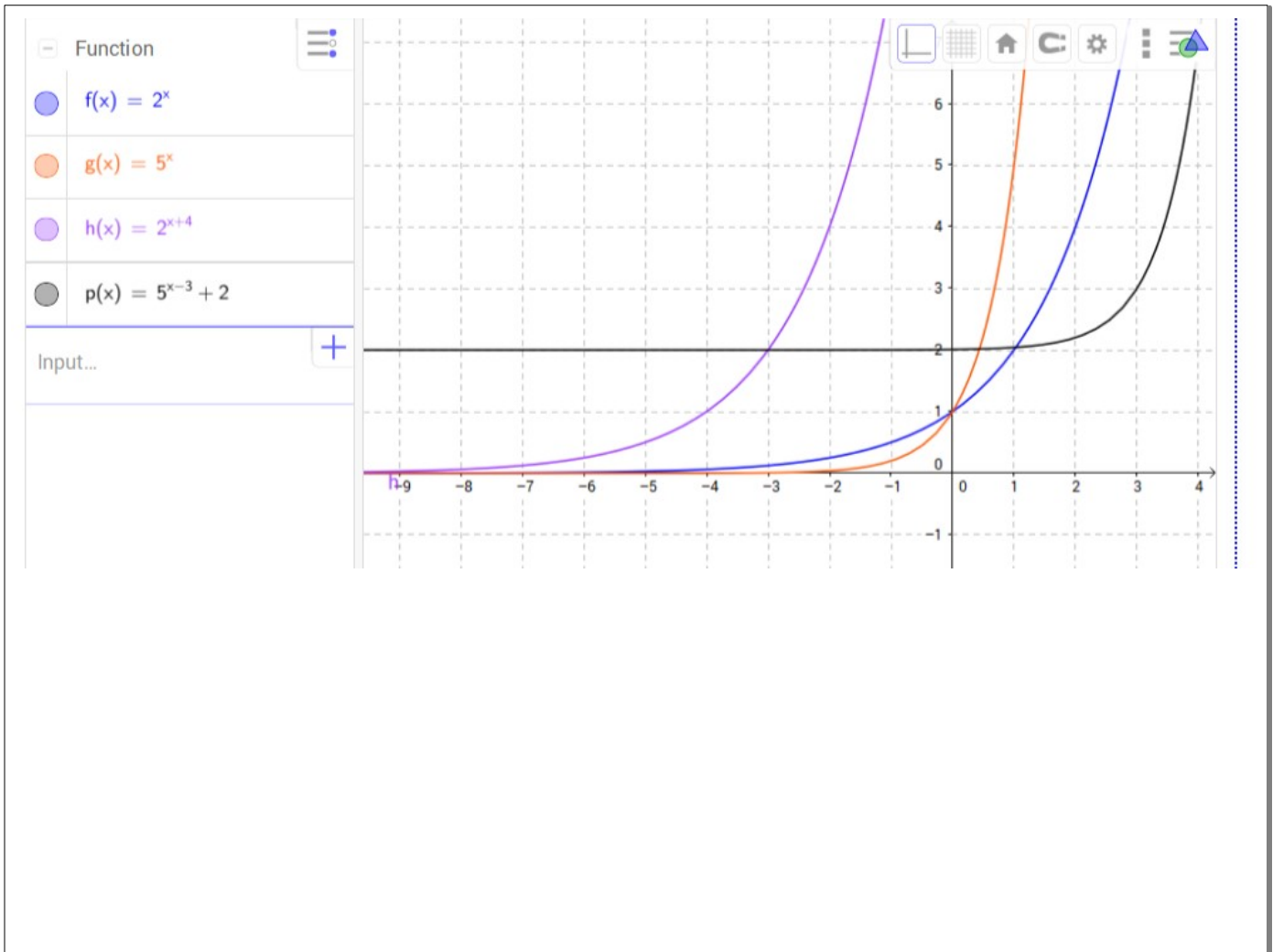
$$b = 1+r, r = \frac{y_2 - y_1}{y_1}$$

a stretch or compresses, reflection across x-axis (multiply the y-values 1st)

h moves left or right (change the sign, add/subtract to the x-values)

k moves up or down (add/subtract to the y-values)

Reminder: to get rid of a negative exponent, the number flips.



Properties of Exponential Functions

Graph each function as a transformation of its parent function.

1. $y = 2^{x+1}$

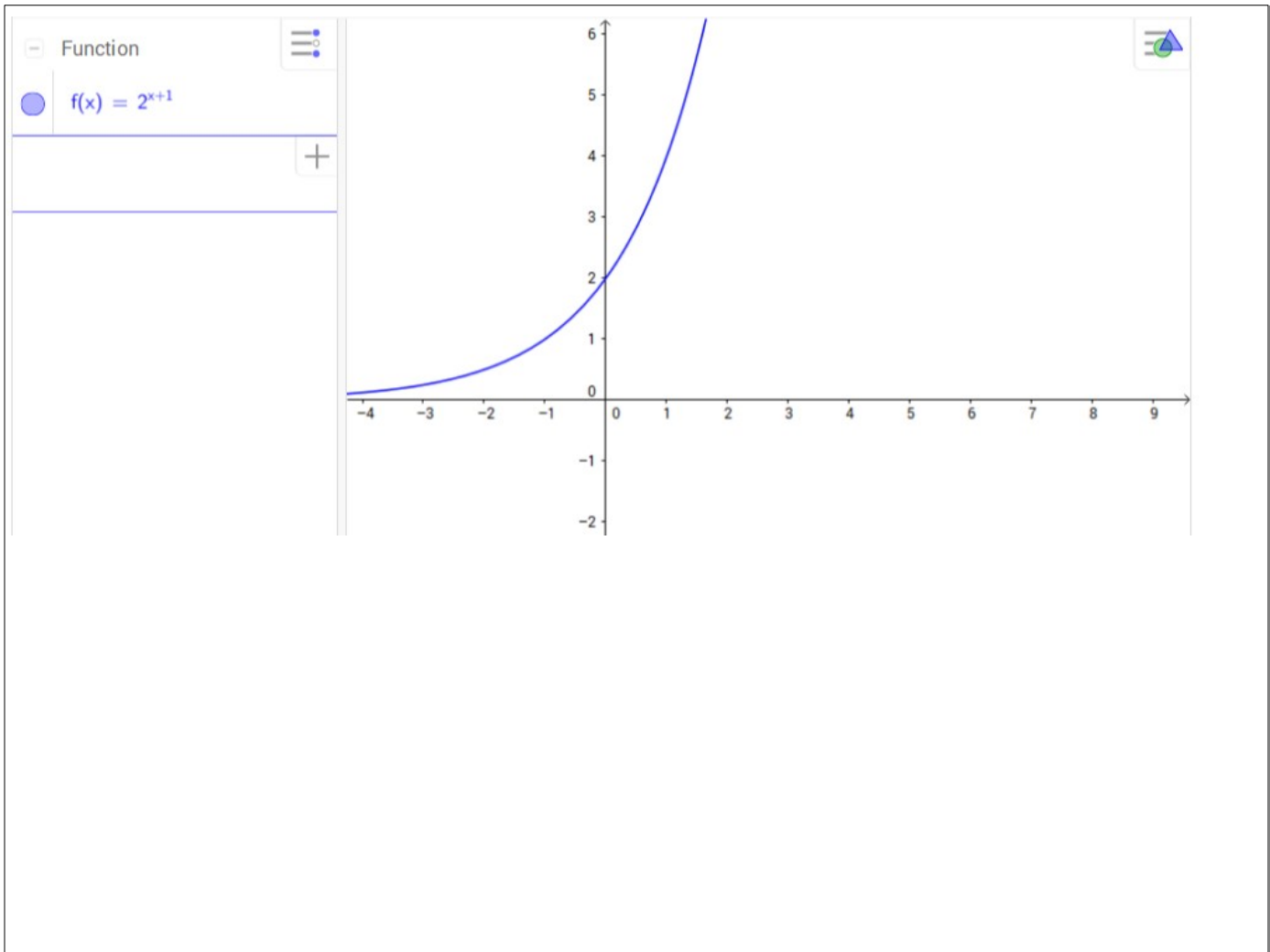
$y = 2^x$

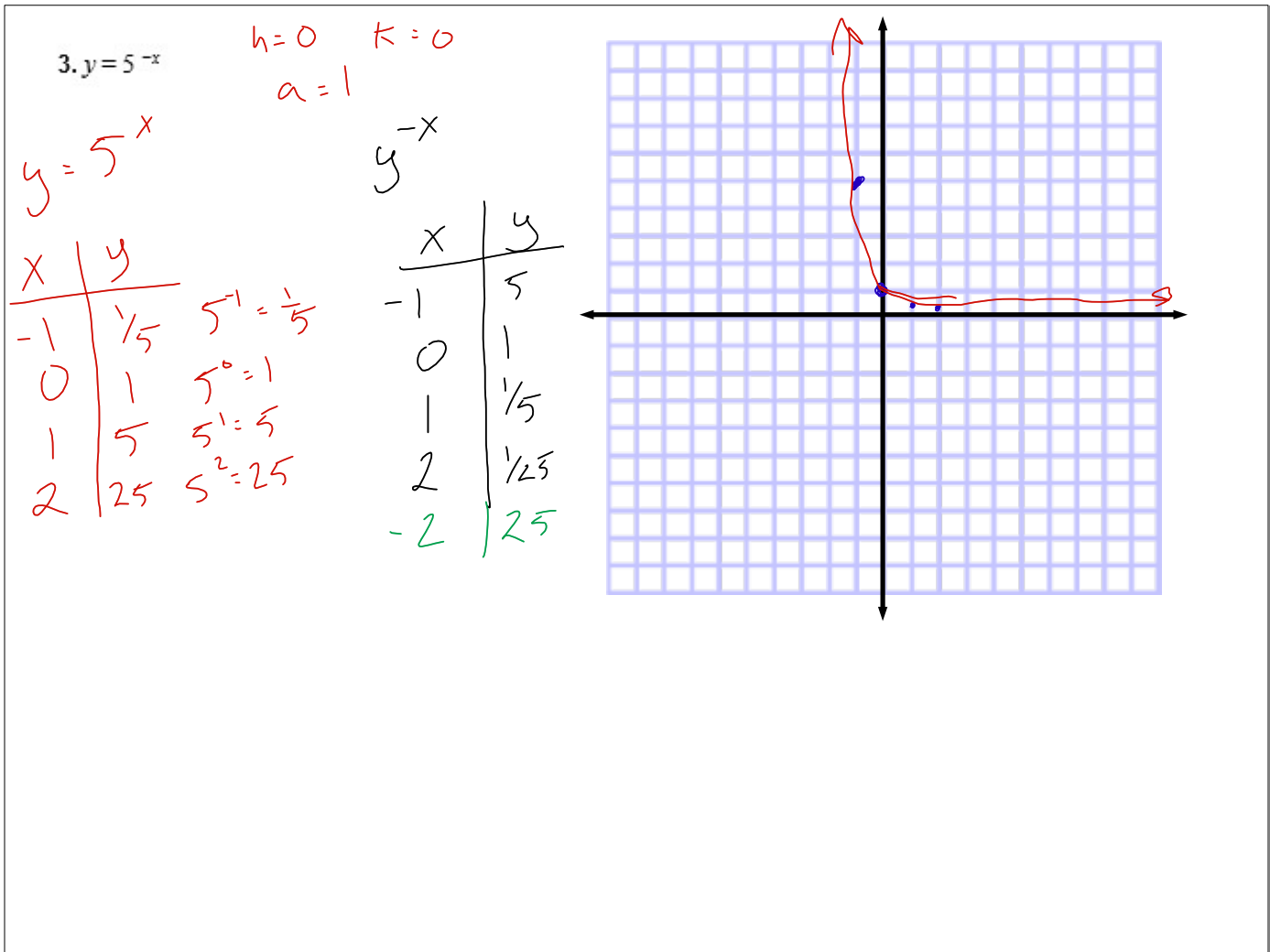
x	y
-1	$\frac{1}{2}$
0	1
1	2
2	4

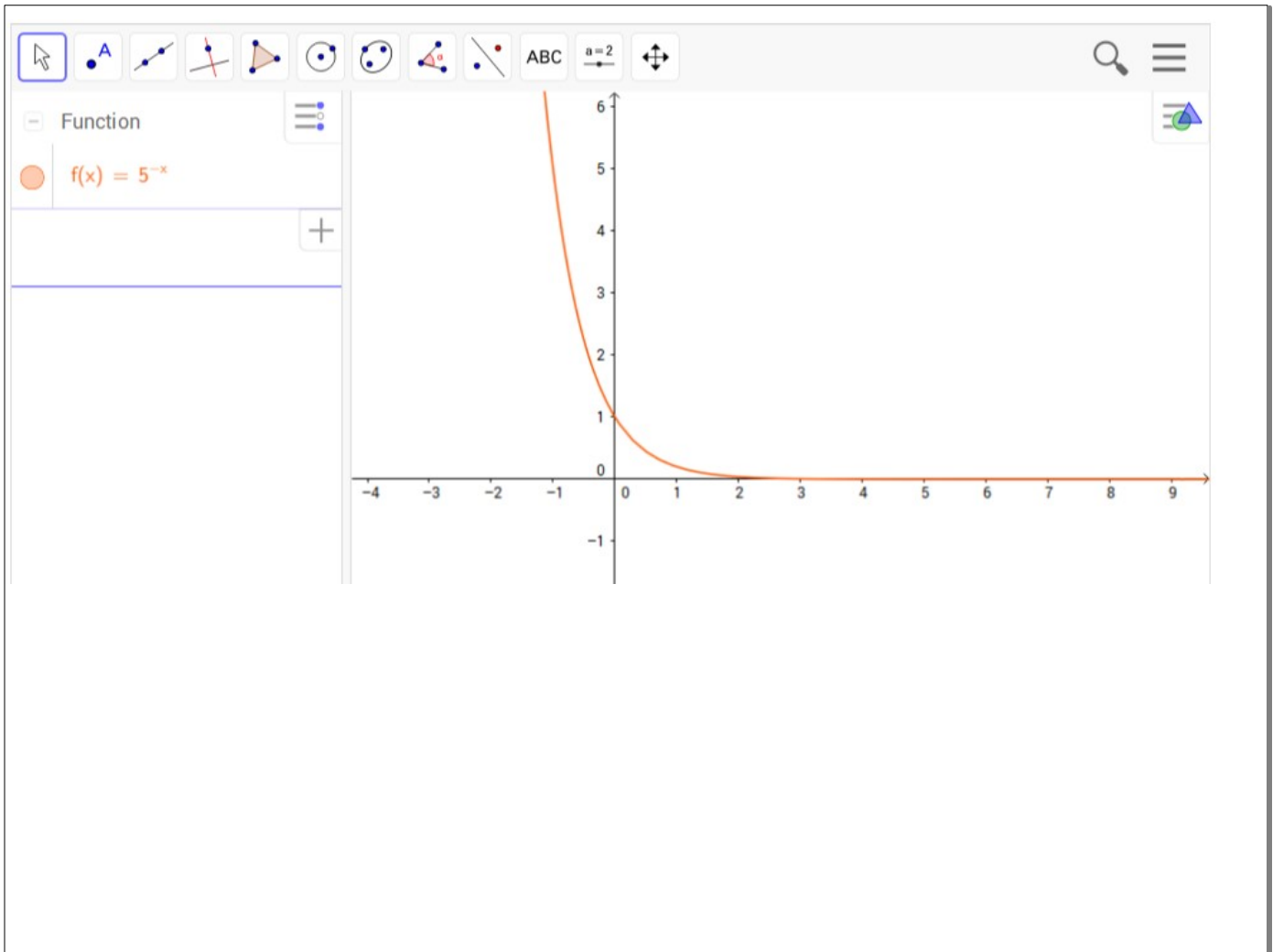
$2^{-1} = \frac{1}{2}$
 $2^0 = 1$
 $2^1 = 2$
 $2^2 = 4$

$y = 2^{x+1}$
 $h = -1$

x-1	y
-2	$\frac{1}{2}$
-1	1
0	2
1	4







5. $y = 2(2)^{x+2}$

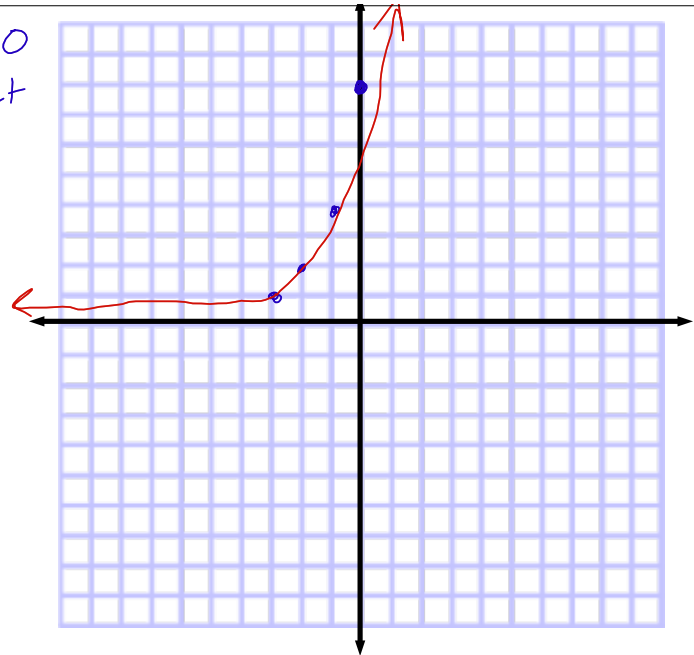
$a = 2$ $h = -2$ $k = 0$
 multiply y -values subtract from x

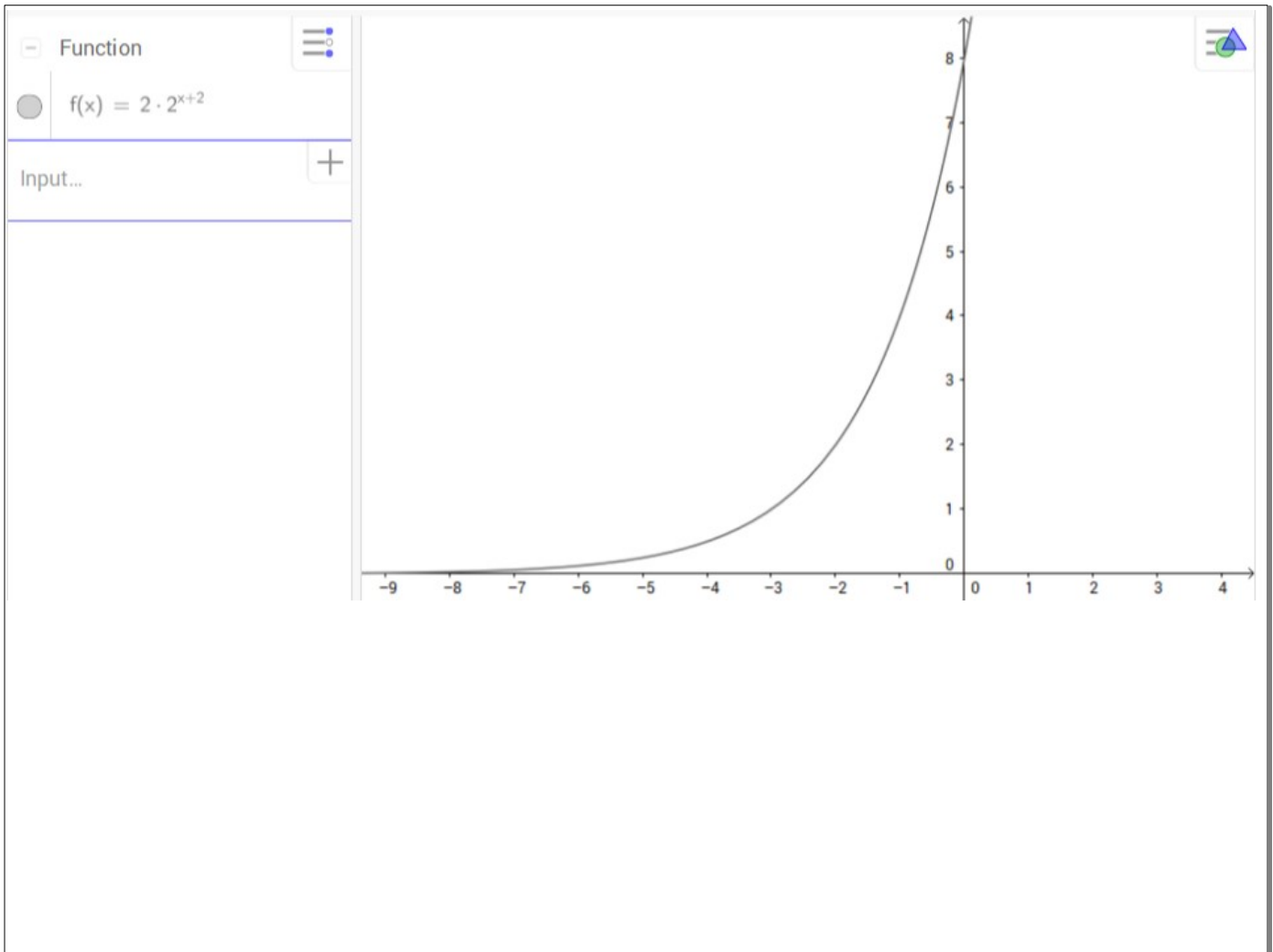
$y = 2(2)^{x+2}$

Parent
 $y = 2^x$

x	y	
-1	$\frac{1}{2}$	$2^{-1} = \frac{1}{2}$
0	1	$2^0 = 1$
1	2	$2^1 = 2$
2	4	$2^2 = 4$

$x-2$	$2y$
-3	1
-2	2
-1	4
0	8

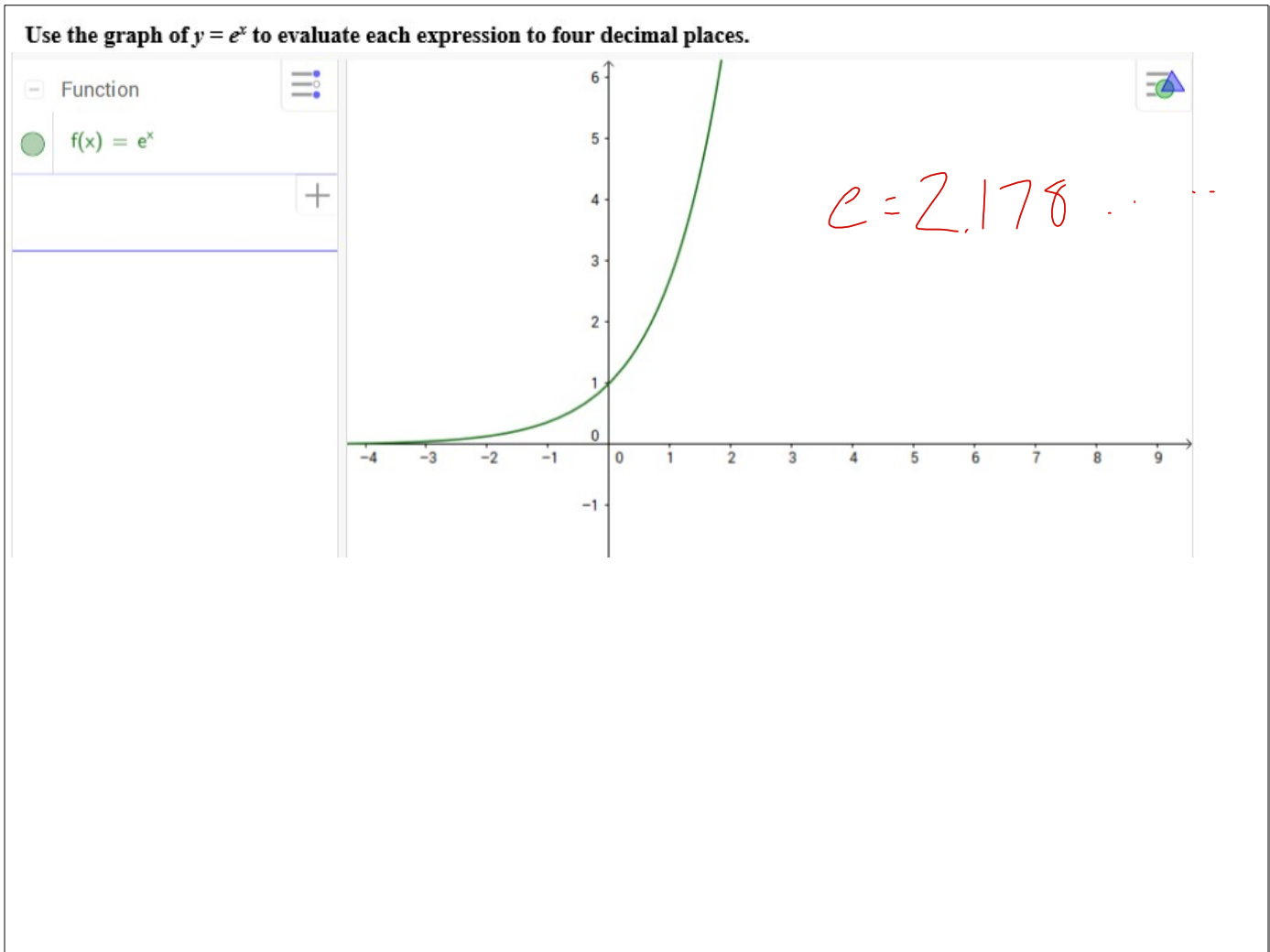




7. A cake is 190°F when you remove it from the oven. You must let it cool to 75°F before you can frost it. The table at the right shows the temperature readings for the cake.

- a. Given a room temperature of 68°F , what is an exponential model for this data set?
- b. How long must the cake cool before you can frost it?

Time (min)	Temp ($^{\circ}\text{F}$)
0	190
5	149
10	122
15	104
20	92



9. $e^{-2.5}$

in calculator

2.5, neg, 2nd, LN

$$e^{-2.5} = .082 \dots$$

Continuously Compounded Interest

$$A(t) = Pe^{rt}$$

A(t) amount after time

P = Principal (beginning amount)

e = number (2.718...)

r = annual interest rate

t = time in years (must convert to years if given months)

Find the amount in a continuously compounded account for the given conditions.

11. principal: \$5000

annual interest rate: 6.9%

time: 30 yr

P

$$r = .069$$

$$t = 30$$

$$A(30) = 5000 e^{(.069)(30)}$$

$$.069 \times 30 = 2.07 \leftarrow \text{In calculator}$$

$$2^{\text{nd}}, \text{LN} \rightarrow 7.924 \dots$$

$$\times 5000 \rightarrow 39624.115$$

$$A(30) = \$39,624.12$$

13. How long would it take to double your principal at an annual interest rate of 7% compounded continuously? $t = ?$

$$P = 5000 \quad r = .07$$

$$A(t) = 10000 \quad t = ?$$

$$\frac{10000}{5000} = \frac{5000 e^{.07t}}{5000}$$

$$2 = e^{.07t}$$

Trial and error

$$2 \neq e^{.07(20)} = 4.055$$

$$2 \neq e^{.07(15)} = 2.85$$

$$2 = e^{.07(10)} = 2.013$$

$$2 \neq e^{.07(9)} = 1.878$$

just under 10 years

15. The isotope Hg-197 is used in kidney scans. It has a half-life of 64.128 h. After that time, half the isotope will have decayed. Write the exponential decay function for a 12-mg sample. Find the amount remaining after 72 h.

$$A = a b^t \quad a = 12$$

$$b = 1 + r$$

$$r = -\frac{1}{2}$$

$$A = 12 \left(\frac{1}{2}\right)^{\frac{h}{64.128}}$$

$$b = 1 - \frac{1}{2} = \frac{1}{2}$$

$$A = 12 \left(\frac{1}{2}\right)^{\frac{72}{64.128}} = 5.5$$

5.5 mg of Hg-197 left after 72 hours.

17. Suppose you invest \$2000 at an annual interest of 5.5% compounded continuously.

- How much will you have in the account in 10 years?
- How long will it take for the account to reach \$5000?

$$A(t) = P e^{rt} \quad P = 2000 \quad r = .055$$

a) $t = 10$ $A(t) = ?$

$$A(t) = 2000 e^{(.055)(10)}$$

$$2000 e^{.55} \quad (2^{\text{nd}} \rightarrow \text{LN})$$

$$2000(1.7\dots)$$

$$3466.506\dots$$

$$A(10) = \$3466.51$$

b) $A(t) = 5000$ trial and error to find t .

$$\frac{5000}{2000} = \frac{2000 e^{.055t}}{2000} \rightarrow 2.5 = e^{.055t}$$

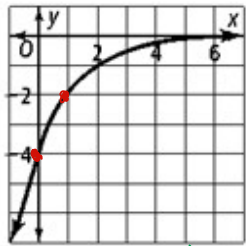
$$t = 16 \quad e^{.055(16)} = 2.4$$

$$t = 17 \quad e^{.055(17)} = 2.55$$

Approximate by 16 and a half years

The parent function for each graph below is of the form $y = ab^x$. Write the parent function. Then write a function for the translation indicated.

19.



translation: right 3 units, up 1 units

$h=3$ $k=1$

$$y = -4\left(\frac{1}{2}\right)^{x-3} + 1$$

x_1, y_1 and x_2, y_2
 $(0, -4)$ and $(1, -2)$

$$b = |1+r|$$

$$r = \frac{y_2 - y_1}{y_1}$$

$$r = \frac{-2 - (-4)}{-4} = \frac{2}{-4} = -\frac{1}{2}$$

$$b = 1 - \frac{1}{2} = \frac{1}{2}$$

Parent: $y = \left(\frac{1}{2}\right)^x$

Function we need

x	y
0	1
1	$\frac{1}{2}$

$$\begin{aligned} |a| &= -4 \\ a &= -4 \end{aligned}$$

x	y
0	-4
1	-2

