Exploring Exponential Models

Make sure you have a calculator.

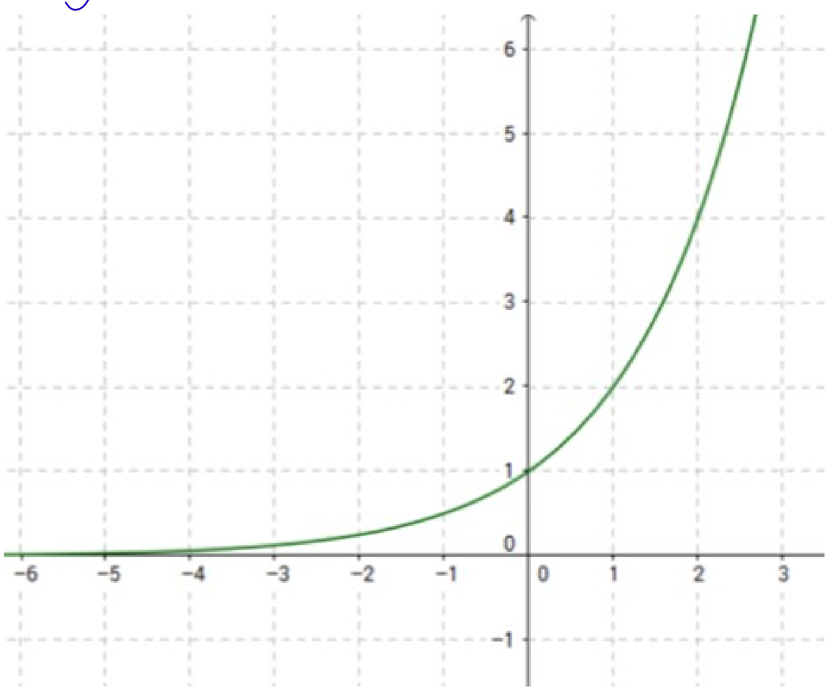
An exponential function looks like this:

$$y = ab^x$$

a cannot equal 0

b must be a positive number, other than 1

$y = 2^x$



Exploring Exponential Models

Graph each function.

1. $y = (0.3)^x$

x	y
-2	11.11
-1	3.33
0	1
1	.3
2	.09

$y = .3^{-1} = \frac{1}{.3} = 3.33$

$y = .3^{0} = 1$

$y = .3^{1} = .3$

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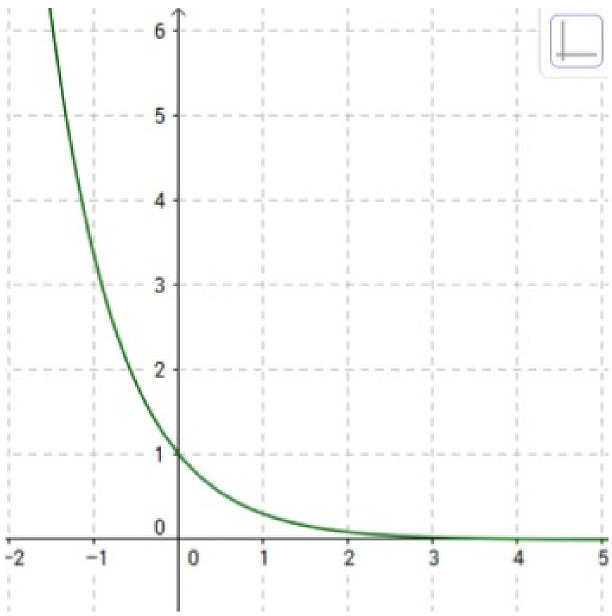

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3. $y = 2(\frac{1}{5})^x$

x	y
-2	50
-1	10
0	2
1	.4
2	.08

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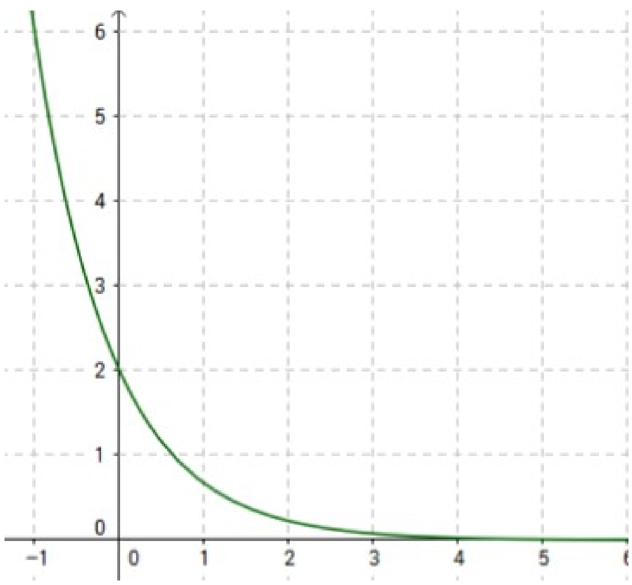
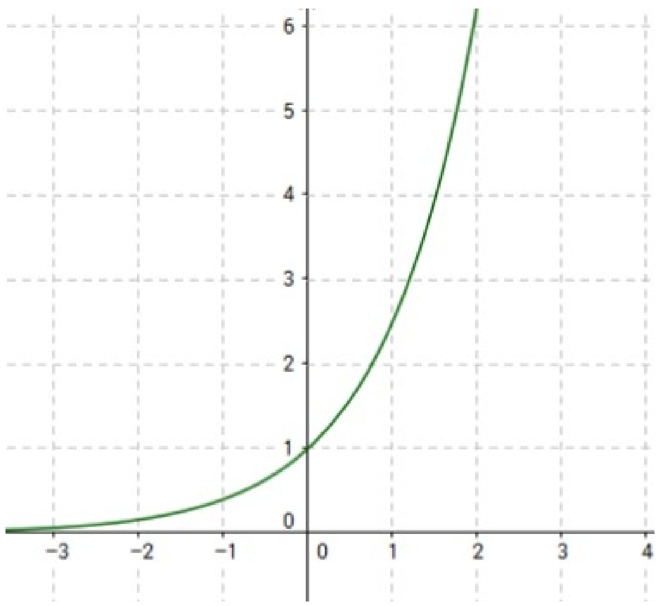

6
5
4
3
2
1
0
−1
0
1
2
3
4
5

5. $s(t) = 2.5^t$

t x	$s(t)$ y
-2	.16
-1	.4
0	1
1	2.5
2	6.25


6
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Exponential growth: as x goes up, y goes up

Exponential decay: as x goes up, y goes down

There is an asymptote at the x-axis

(0, a) is the y-intercept

As long as a is positive:

$b>1$, exponential growth, b is the growth factor

$0<b<1$ (a fraction), exponential decay, b is the decay factor

$y=0$ is the asymptote and range is $y>0$

Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the y-intercept.

7. $y = 0.99\left(\frac{1}{3}\right)^x$

decay

decay factor: $\frac{1}{3}$

y-int: (0, .99)

9. $y = 185\left(\frac{5}{4}\right)^x$

growth

y-int: (0, 185)

11. $f(x) = 0.25(1.05)^x$

Growth

y-int: (0, .25)

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Creating equations for exponential growth and decay:

For growth or decay to be exponential, the amount must change by a fixed percentage each time period

$$A = a(1+r)^t$$

$b = 1 + r$

growth/decay factor

A is amount after t, time period.

a is the initial amount

r is the rate of growth or decay

t is the number of time periods

1 + r is the growth or decay factor

13. Suppose you deposit \$1500 in a savings account that pays interest at an annual rate of 6%. No money is added or withdrawn from the account.

a. How much will be in the account after 5 years?

b. How much will be in the account after 20 years?

c. How many years will it take for the account to contain \$2500?

d. How many years will it take for the account to contain \$4000?

$a = 1500$ $r = .06$ $t =$ years

$A = 1500(1 + .06)^t$

$A = 1500(1.06)^t$

a) $A = 1500(1.06)^5$ $A = \$2007.34$

b) $A = 1500(1.06)^{20}$ $A = \$4810.71$

c) $2500 = 1500(1.06)^t$ trial and error

$t = 10$ $A = 2686.27$

$t = 9$ $A = 2534.22$

A little under 9 years

Write an exponential function to model each situation. Find each amount after the specified time.

$A = 583448$

a $r = -.014$ $t = 18$

15. A population of 752,000 decreases 1.4% per year for 18 years.

$A = 752{,}000(.986)^t$

function

$A = 752000(.986)^{18}$

$A = 583448.207$

For each annual rate of change, find the corresponding growth or decay factor.

$b = 1 + r$

17. + 45%

$r = .45$

$1 + .45 = 1.45$

19. − 40%

$r = -.4$

$1 - .4 = .6$

21. + 28%

$r = .28$

$1 + .28 = 1.28$

23. − 5%

$r = -.05$

$1 - .05 = .95$

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The growth or decay factor is 1+r.

If you do not have r, and they give you points, then $r = \dfrac{y_2 - y_1}{y_1}$

The order **does** matter.

25. In 2009, there were 1570 bears in a wildlife refuge. In 2010, the population had increased to approximately 1884 bears. If this trend continues and the bear population is increasing exponentially, how many bears will there be in 2015?

$r = \frac{1884 - 1570}{1570} = .2$

$b = 1 + .2 = 1.2$

$a = 1570, \ t = 0$

$A = 1570(1.2)^t$

2015

$t = 2015 - 2009 = 6$

$A = 1570(1.2)^6$

$A = 4687.99$

4688 bears by 2015

27. Your friend drops a rubber ball from 4 ft. You notice that its rebound is 32.5 in. on the first bounce and 22 in. on the second bounce.

a. What exponential function would be a good model for the height of the ball?

b. How high will the ball bounce on the fourth bounce?

a) $r = \frac{22 - 32.5}{32.5} = -.323$

$a = 4ft = 48in$ $b = 1 - .323 = .677$

$A = 48(.677)^t$

b) $A = 48(.677)^4$

$A = 10.08 \text{ in}$

29. A new truck that sells for $29,000 depreciates 12% each year. What is the value of the truck after 7 years?

$a = 29000$ $r = .12$ $b = 1 - .12 = .88$

$A = 29000(.88)^t$

$A = 29000(.88)^7$

$A = 11,851.59$

$\$11,851.59$

31. The population of an endangered bird is decreasing at a rate of 0.75% per year. There are currently about 200,000 of these birds.

a. What exponential function would be a good model for the population of these endangered birds?

b. How many birds will there be in 100 years?

a) $r = -.0075$ $b = 1 - .0075 = .9925$

$A = 200000(.9925)^t$

b) $A = 200000(.9925)^{100}$

$A = 94206.65 \approx 94207$ birds

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