Pre-Calc Final Exam Review Name_____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Find the exact value of the expression.

1) $\cos\left(\frac{2\pi}{9} - \frac{\pi}{18}\right)$ 1) _____

A) $\frac{\sqrt{3}}{2}$ B) $\frac{1}{4}$ C) 1 D) $\frac{1}{2}$

Find the exact value by using a sum or difference identity.

2) $\sin(200° - 80°)$ 2) _____

A) $\frac{\sqrt{3}}{2}$ B) $-\frac{10}{3}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{1}{2}$

Find the exact value of the expression.

3) $\sin 255° \cos 15° - \cos 255° \sin 15°$ 3) _____

A) $\frac{\sqrt{3}}{2}$ B) $\frac{17}{4}$ C) $-\frac{\sqrt{3}}{2}$ D) $-\frac{1}{2}$

Identify α and β in the following expression which is the right side of the formula for $\cos(\alpha - \beta)$.

4) $\cos(155°)\cos(35°) + \sin(155°)\sin(35°)$ 4) _____

A) $\alpha = 35°, \beta = 155°$ B) $\alpha = 155°, \beta = 35°$

C) $\alpha = -155°, \beta = 35°$ D) $\alpha = -35°, \beta = 155°$

Use the given information to find the exact value of the expression.

5) $\sin\alpha = \frac{5}{13}$, α lies in quadrant II, and $\cos\beta = \frac{15}{17}$, β lies in quadrant I Find $\sin(\alpha - \beta)$. 5) _____

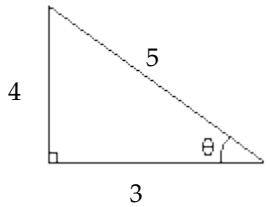
A) $\frac{220}{221}$ B) $\frac{171}{221}$ C) $\frac{140}{221}$ D) $-\frac{21}{221}$

6) $\sin\alpha = \frac{4}{5}$, α lies in quadrant II, and $\cos\beta = \frac{2}{5}$, β lies in quadrant I Find $\cos(\alpha - \beta)$. 6) _____

A) $\frac{6 - 4\sqrt{21}}{25}$ B) $\frac{8 + 3\sqrt{21}}{25}$ C) $\frac{-6 + 4\sqrt{21}}{25}$ D) $\frac{8 - 3\sqrt{21}}{25}$

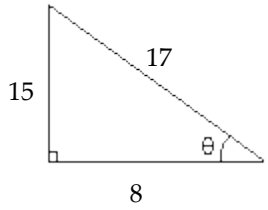
Use the figure to find the exact value of the trigonometric function.

7) Find $\cos 2\theta$. 7) _____



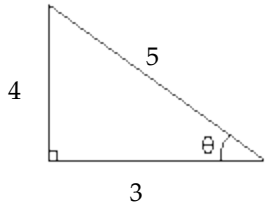
A) $\frac{7}{25}$ B) $-\frac{7}{25}$ C) $\frac{24}{25}$ D) $-\frac{1}{5}$

8) Find $\tan 2\theta$. 8) _____



A) $\frac{160}{161}$ B) $-\frac{240}{161}$ C) $-\frac{161}{289}$ D) $\frac{240}{289}$

9) Find $\sin 2\theta$. 9) _____



A) $\frac{7}{25}$ B) $\frac{24}{25}$ C) $-\frac{7}{25}$ D) $-\frac{8}{25}$

Use the given information to find the exact value of the expression.

10) $\cos\theta = \frac{5}{13}$, θ lies in quadrant IV Find $\sin 2\theta$. 10) _____

A) $\frac{119}{169}$ B) $-\frac{119}{169}$ C) $\frac{120}{169}$ D) $-\frac{120}{169}$

Use the given information to find the exact value of the trigonometric function.

11) $\sin\theta = \frac{1}{4}$, $\tan\theta > 0$ Find $\cos\frac{\theta}{2}$. 11) _____

A) $\frac{\sqrt{10}}{4}$ B) $\frac{\sqrt{8-2\sqrt{15}}}{4}$ C) $\frac{\sqrt{8+2\sqrt{15}}}{4}$ D) $\frac{\sqrt{6}}{4}$

12) $\sin\theta = \frac{1}{4}$, θ lies in quadrant I Find $\sin\frac{\theta}{2}$. 12) _____

A) $\frac{\sqrt{8+2\sqrt{15}}}{4}$ B) $\frac{\sqrt{8-2\sqrt{15}}}{4}$ C) $\frac{\sqrt{6}}{4}$ D) $\frac{\sqrt{10}}{4}$

Find all solutions of the equation.

13) $2\cos x - 1 = 0$ 13) _____

A) $x = \frac{4\pi}{3} + 2n\pi$ or $x = \frac{5\pi}{6} + 2n\pi$ B) $x = \frac{\pi}{3} + 2n\pi$ or $x = \frac{5\pi}{3} + 2n\pi$

C) $x = \frac{\pi}{3} + n\pi$ or $x = \frac{5\pi}{3} + n\pi$ D) $x = \frac{4\pi}{3} + n\pi$ or $x = \frac{5\pi}{6} + n\pi$

Solve the equation on the interval [0, 2π).

14) $\sin 4x = \frac{\sqrt{3}}{2}$ 14) _____

A) $\frac{\pi}{12}, \frac{\pi}{6}, \frac{2\pi}{3}, \frac{7\pi}{12}, \frac{7\pi}{6}, \frac{13\pi}{12}, \frac{5\pi}{3}, \frac{19\pi}{12}$ B) $\frac{\pi}{4}, \frac{5\pi}{4}$

C) 0 D) $0, \frac{\pi}{4}, \pi$

15) $\sin^2 x + \sin x = 0$ 15) _____

A) $0, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$ B) $0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}$ C) $0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$ D) $0, \pi, \frac{3\pi}{2}$

Solve the equation on the interval [0, 2π).

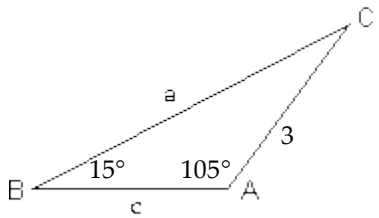
16) $\cos x + 2 \cos x \sin x = 0$ 16) _____

A) $0, \frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}$ B) $\frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi$

C) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ D) $\frac{7\pi}{6}, \frac{11\pi}{6}$

Solve the triangle.

17) 17) _____



A) C = 60°, a = 10.04, c = 11.2 B) C = 60°, a = 11.2, c = 10.04

C) C = 55°, a = 10.04, c = 11.2 D) C = 65°, a = 11.2, c = 10.04

Two sides and an angle (SSA) of a triangle are given. Determine whether the given measurements produce one triangle, two triangles, or no triangle at all. Solve each triangle that results. Round lengths to the nearest tenth and angle measures to the nearest degree.

18) B = 17°, b = 13.8, a = 15.73 18) _____

A) $A_1 = 19°, C_1 = 144°, c_1 = 27.7$; $A_2 = 161°, C_2 = 2°, c_2 = 1.6$ B) no triangle

C) A = 161°, C = 2°, c = 1.6 D) A = 19°, C = 144°, c = 27.7

Find the area of the triangle having the given measurements. Round to the nearest square unit.

19) A = 27°, b = 14 inches, c = 5 inches 19) _____

A) 16 square inches B) 14 square inches

C) 33 square inches D) 31 square inches

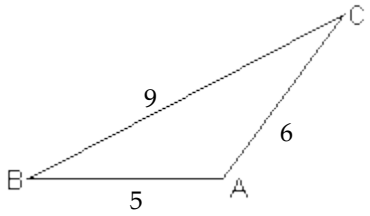
Solve the problem.

20) Two tracking stations are on the equator 173 miles apart. A weather balloon is located on a bearing of N36°E from the western station and on a bearing of N18°W from the eastern station. How far is the balloon from the western station? Round to the nearest mile. 20) _____

A) 173 miles B) 203 miles C) 164 miles D) 212 miles

Solve the triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

21) 21) _____



A) A = 109°, B = 39°, C = 32° B) A = 39°, B = 32°, C = 109°

C) A = 39°, B = 109°, C = 32° D) A = 109°, B = 32°, C = 39°

Solve the problem.

22) Two airplanes leave an airport at the same time, one going northwest (bearing 135°) at 417 mph and the other going east at 329 mph. How far apart are the planes after 4 hours (to the nearest mile)? 22) _____

A) 2300 miles B) 2170 miles C) 690 miles D) 2760 miles

Use Heron's formula to find the area of the triangle. Round to the nearest square unit.

23) a = 8 meters, b = 13 meters, c = 12 meters 23) _____

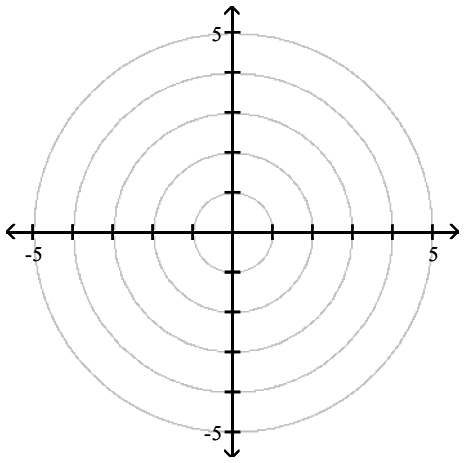
A) 98 square meters B) 100 square meters

C) 49 square meters D) 25 square meters

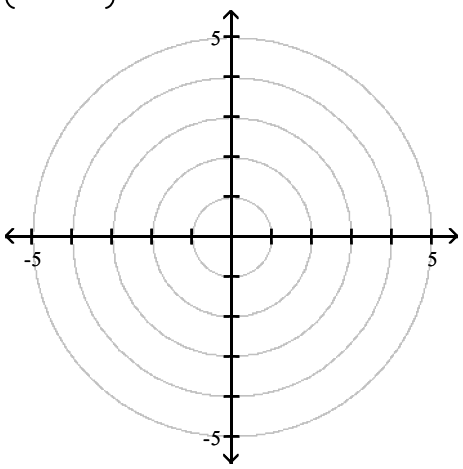
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Use a polar coordinate system to plot the point with the given polar coordinates.

24) $\left(2, \frac{9\pi}{4}\right)$ 24) _____

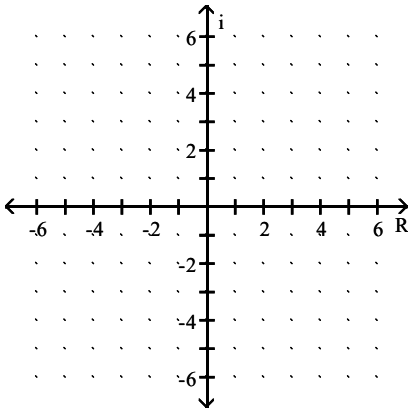


25) $\left(-4, \frac{-5\pi}{4}\right)$ 25) _____

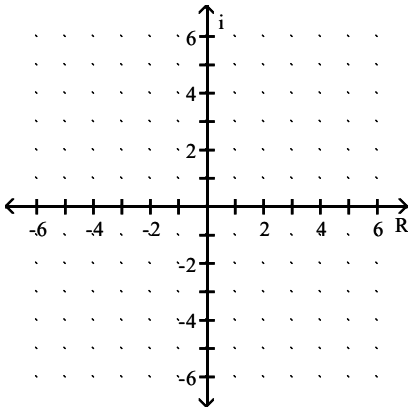


Plot the complex number.

26) –1 – 2i 26) _____

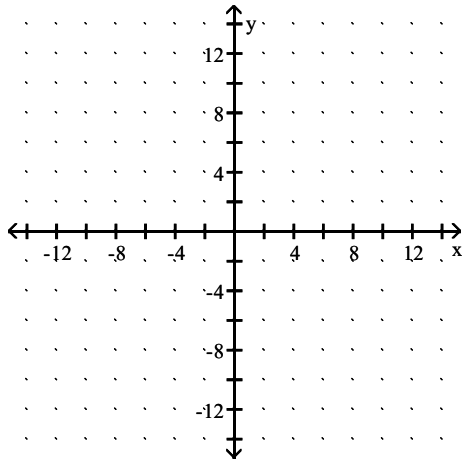


27) –4 + i 27) _____



Sketch the vector as a position vector and find its magnitude.

28) $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ 28) _____



Find the modulus and the argurment of the complex number.

29) $z = -3 + 6i$ 29) _____

30) $z = -15 + 4i$ 30) _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Polar coordinates of a point are given. Find the rectangular coordinates of the point.

31) $\left(7, \frac{2\pi}{3}\right)$ 31) _____

A) $\left(-\frac{7}{2}, \frac{-7\sqrt{3}}{2}\right)$ B) $\left(\frac{7}{2}, \frac{-7\sqrt{3}}{2}\right)$ C) $\left(\frac{7}{2}, \frac{7\sqrt{3}}{2}\right)$ D) $\left(-\frac{7}{2}, \frac{7\sqrt{3}}{2}\right)$

32) $(-3, -135°)$ 32) _____

A) $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$ B) $\left(\frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$ C) $\left(\frac{-3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$ D) $\left(\frac{3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$

The rectangular coordinates of a point are given. Find polar coordinates of the point. Express θ in radians.

33) $(6, -6\sqrt{3})$ 33) _____

A) $\left(6, \frac{5\pi}{3}\right)$ B) $\left(6, \frac{11\pi}{6}\right)$ C) $\left(12, \frac{11\pi}{6}\right)$ D) $\left(12, \frac{5\pi}{3}\right)$

34) $(-5, 0)$ 34) _____

A) $(5, \pi)$ B) $\left(5, \frac{3\pi}{2}\right)$ C) $(5, 0)$ D) $\left(5, \frac{\pi}{2}\right)$

Find the specified vector or scalar.

35) $\mathbf{u} = -7\mathbf{i} - 3\mathbf{j}$, $\mathbf{v} = -5\mathbf{i} + 7\mathbf{j}$; Find $\mathbf{u} + \mathbf{v}$. 35) _____

A) $-13\mathbf{i} + 4\mathbf{j}$ B) $-2\mathbf{i} - 13\mathbf{j}$ C) $-12\mathbf{i} + 4\mathbf{j}$ D) $2\mathbf{i} + 4\mathbf{j}$

36) $\mathbf{u} = -9\mathbf{i} - 2\mathbf{j}$, $\mathbf{v} = 5\mathbf{i} + 7\mathbf{j}$; Find $\mathbf{u} - \mathbf{v}$. 36) _____

A) $-16\mathbf{i} + 5\mathbf{j}$ B) $-14\mathbf{i} - 9\mathbf{j}$ C) $-15\mathbf{i} + 5\mathbf{j}$ D) $-4\mathbf{i} + 5\mathbf{j}$

37) $\mathbf{v} = -7\mathbf{i} + 2\mathbf{j}$; Find $\|9\mathbf{v}\|$. 37) _____

A) $27\sqrt{5}$ B) $27i\sqrt{5}$ C) $-9\sqrt{53}$ D) $9\sqrt{53}$

Write the vector v in terms of i and j whose magnitude $\|\mathbf{v}\|$ and direction angle θ are given.

38) $\|\mathbf{v}\| = 10,\ \theta = 120°$ 38) _____

A) $\mathbf{v} = -5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}$ B) $\mathbf{v} = 5\mathbf{i} - 5\sqrt{3}\mathbf{j}$

C) $\mathbf{v} = -5\mathbf{i} + 5\sqrt{3}\mathbf{j}$ D) $\mathbf{v} = 5\sqrt{3}\mathbf{i} - 5\mathbf{j}$

Write the standard form of the equation of the circle with the given center and radius.

39) (0, 9); 5 39) _____

A) $x^2 + (y - 9)^2 = 25$ B) $(x - 9)^2 + y^2 = 25$

C) $(x + 9)^2 + y^2 = 25$ D) $x^2 + (y + 9)^2 = 5$

Complete the square and write the equation in standard form. Then give the center and radius of the circle.

40) $x^2 + y^2 + 6x - 2y + 10 = 36$ 40) _____

A) $(x - 1)^2 + (y + 3)^2 = 36$
$(-1, 3)$, $r = 36$

B) $(x + 3)^2 + (y - 1)^2 = 36$
$(3, -1)$, $r = 36$

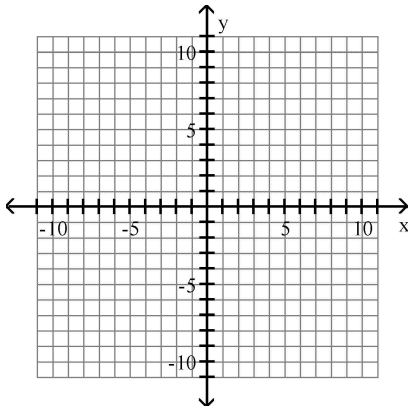
C) $(x + 3)^2 + (y - 1)^2 = 36$
$(-3, 1)$, $r = 6$

D) $(x - 1)^2 + (y + 3)^2 = 36$
$(1, -3)$, $r = 6$

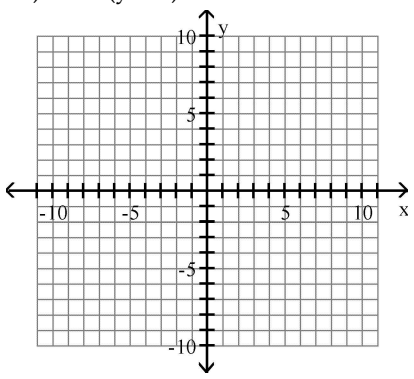
SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Graph the ellipse.

41) $\frac{(x - 1)^2}{16} + \frac{(y + 1)^2}{9} = 1$ 41) _____

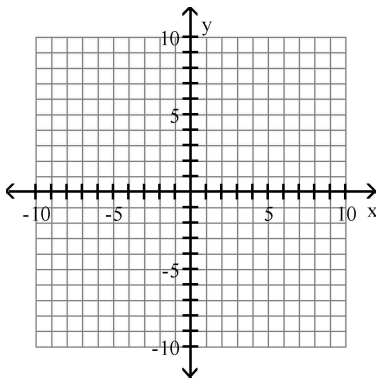


42) $4(x - 1)^2 + 16(y + 1)^2 = 64$ 42) _____



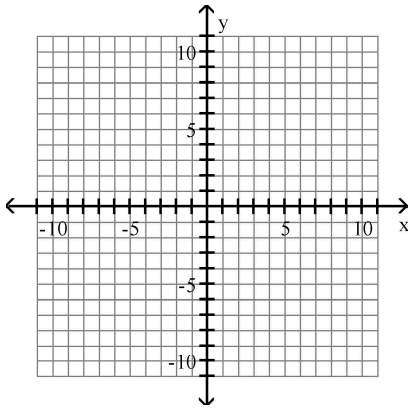
Graph the parabola with the given equation.

43) $(y + 2)^2 = 6(x - 2)$ 43) _____



Use the center, vertices, and asymptotes to graph the hyperbola.

44) $\frac{(x - 2)^2}{4} - \frac{(y + 1)^2}{9} = 1$ 44) _____



MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Convert the equation to the standard form for an ellipse by completing the square on x and y.

45) $4x^2 + 16y^2 - 16x - 96y + 96 = 0$ 45) _____

A) $\frac{(x+2)^2}{16} + \frac{(y+3)^2}{4} = 1$

B) $\frac{(x-2)^2}{4} + \frac{(y-3)^2}{16} = 1$

C) $\frac{(x-3)^2}{16} + \frac{(y-2)^2}{4} = 1$

D) $\frac{(x-2)^2}{16} + \frac{(y-3)^2}{4} = 1$

Find the location of the center, vertices, and foci for the hyperbola described by the equation.

46) $\frac{(x+2)^2}{64} - \frac{(y+4)^2}{100} = 1$ 46) _____

A) Center: (–2, –4); Vertices: (–10, 4) and (6, 4); Foci: $(-2 - 2\sqrt{41}, 4)$ and $(-2 + 2\sqrt{41}, 4)$

B) Center: (2, 4); Vertices: (–6, 4) and (10, 4); Foci: $(2 - 2\sqrt{41}, 4)$ and $(2 + 2\sqrt{41}, 4)$

C) Center: (–2, –4); Vertices: (–9, –4) and (7, –4); Foci: $(-1 + 2\sqrt{41}, -3)$ and $(-3 + 2\sqrt{41}, -3)$

D) Center: (–2, –4); Vertices: (–10, –4) and (6, –4); Foci: $(-2 - 2\sqrt{41}, -4)$ and $(-2 + 2\sqrt{41}, -4)$

47) $\frac{(y+4)^2}{9} - \frac{(x+4)^2}{100} = 1$ 47) _____

A) Center: (–4, –4); Vertices: $(-4, -4 - \sqrt{109})$ and $(-4, -4 + \sqrt{109})$; Foci: (–4, –7) and (–4, –1)

B) Center: (–4, –4); Vertices: (–4, –7) and (–4, –1); Foci: $(-4, -4 - \sqrt{109})$ and $(-4, -4 + \sqrt{109})$

C) Center: (4, 4); Vertices: (4, 1) and (4, 7); Foci: $(4, 4 - \sqrt{109})$ and $(4, 4 + \sqrt{109})$

D) Center: (–4, –4); Vertices: (7, –6) and (–3, 0); Foci: $(7, -3 - \sqrt{109})$ and $(-3, -3 + \sqrt{109})$

Convert the equation to the standard form for a hyperbola by completing the square on x and y.

48) $x^2 - y^2 - 2x - 6y - 9 = 0$ 48) _____

A) $(x-1)^2 - (y+3)^2 = 1$

B) $(y-1)^2 - (x+3)^2 = 1$

C) $\frac{(y-1)^2}{36} - \frac{(x+3)^2}{4} = 1$

D) $(x-1)^2 + (y+3)^2 = 1$

Convert the equation to the standard form for a parabola by completing the square on x or y as appropriate.

49) $y^2 - 4y - 2x - 2 = 0$ 49) _____

A) $(y+2)^2 = 2(x+3)$

B) $(y-2)^2 = 2(x+3)$

C) $(y+2)^2 = -2(x+3)$

D) $(y-2)^2 = 2(x-3)$

Find the standard form of the equation of the parabola using the information given.

50) Focus: (3, 3); Directrix: y = –5 50) _____

A) $(y-3)^2 = 16(x+1)$

B) $(y+1)^2 = 16(x-3)$

C) $(x-3)^2 = 16(y+1)$

D) $(x+1)^2 = 16(y-3)$

Find the vertex, focus, and directrix of the parabola with the given equation.

51) $(y + 2)^2 = 16(x + 1)$ 51) _____

A) vertex: (–1, –2)
focus: (–5, –2)
directrix: $x = 3$

B) vertex: (–1, –2)
focus: (3, –2)
directrix: $x = –5$

C) vertex: (–2, –1)
focus: (2, –1)
directrix: $x = –6$

D) vertex: (1, 2)
focus: (5, 2)
directrix: $x = –3$

52) $(x + 4)^2 = 20(y + 2)$ 52) _____

A) vertex: (–2, –4)
focus: (–2, 1)
directrix: $y = –9$

B) vertex: (4, 2)
focus: (4, 7)
directrix: $y = –3$

C) vertex: (–4, –2)
focus: (–4, –7)
directrix: $x = 3$

D) vertex: (–4, –2)
focus: (–4, 3)
directrix: $y = –7$