Exploring Exponential Models

An exponential function looks like this:

$$y = ab^x$$

a cannot equal 0

b must be a positive number, other than 1

$y = a\,b^{x-h} + k$

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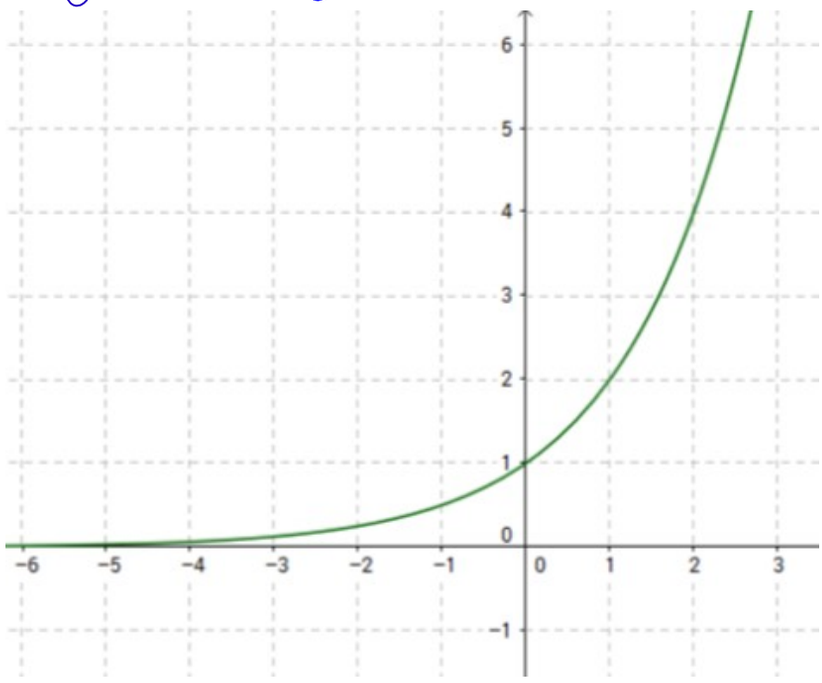
$y = 2^x$ $b = 2$ $a = 1$ $h = 0$ $k = 0$

parent function



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Exploring Exponential Models

Graph each function.

parent function

1. $y = (0.3)^x$ $b = .3$ $a = 1$ $h = 0$ $k = 0$

x	y
-2	11.11
-1	3.3
0	1
1	.3
2	.09

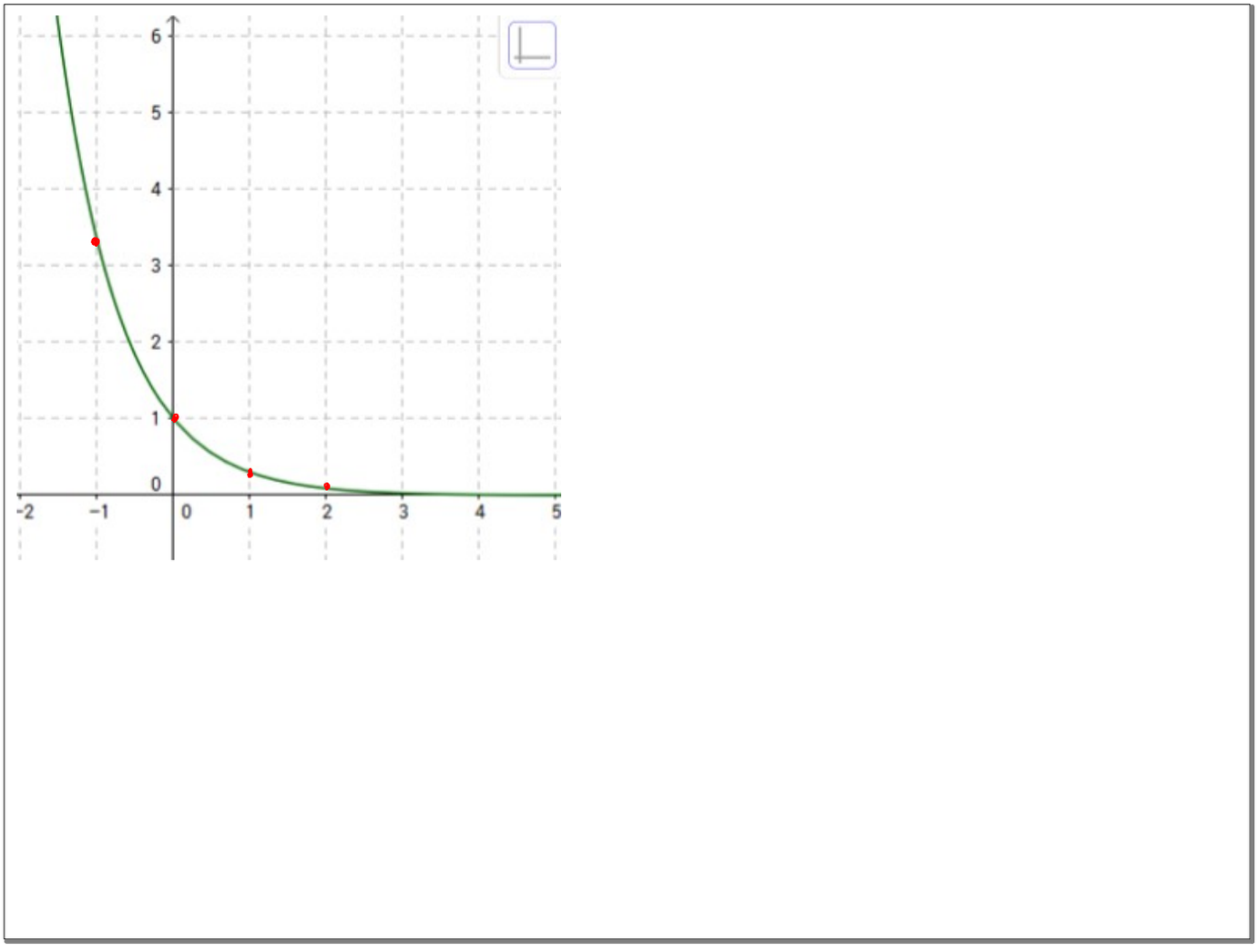
$y = (.3)^{-2} = \frac{1}{.09} = 11.11$

$y = (.3)^{-1} = \frac{1}{.3^1} = \frac{1}{.3} = 3.\overline{3}$

$y = (.3)^0 = 1$

$y = (.3)^1 = .3$

$y = (.3)^2 = .09$



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3. $y = 2(\frac{1}{5})^x$

$b = \frac{1}{5}$ $a = 2$ $h = 0$ $k = 0$

Not parent function

$y = (\frac{1}{5})^x$

x	y
-2	25
-1	5
0	1
1	$\frac{1}{5}$
2	$\frac{1}{25}$

$y = (\frac{1}{5})^{-2} = (\frac{5}{1})^2 = 5^2 = 25$

$y = (\frac{1}{5})^{-1} = (\frac{5}{1})^1 = 5^1 = 5$

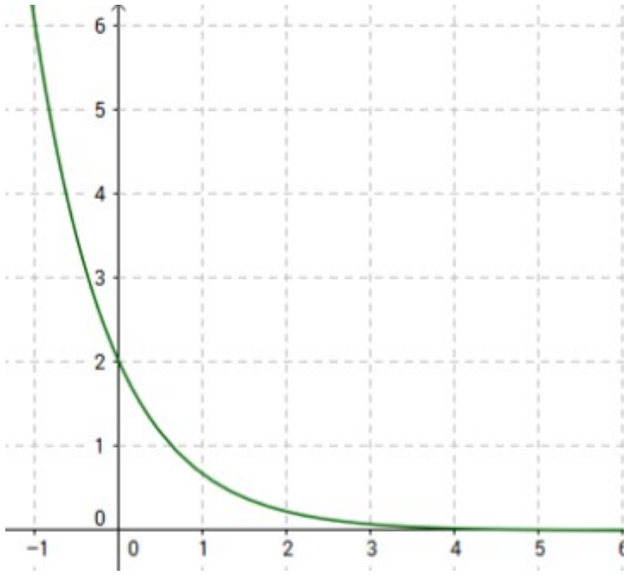
$y = (\frac{1}{5})^0 = 1$

$y = (\frac{1}{5})^1$

$y = (\frac{1}{5})^2 = \frac{1^2}{5^2} = \frac{1}{25}$

x	2(y)
-2	50
-1	10
0	2
1	$\frac{2}{5}$
2	$\frac{2}{25}$

graph



decay

5. $s(t) = 2.5^t$

$b = 2.5$ $a = 1$ $h = 0$ $k = 0$

parent function

x	y
-2	.16
-1	.4
0	1
1	2.5
2	6.25

$y = 2.5^{-2} = \frac{1}{6.25} = .16$

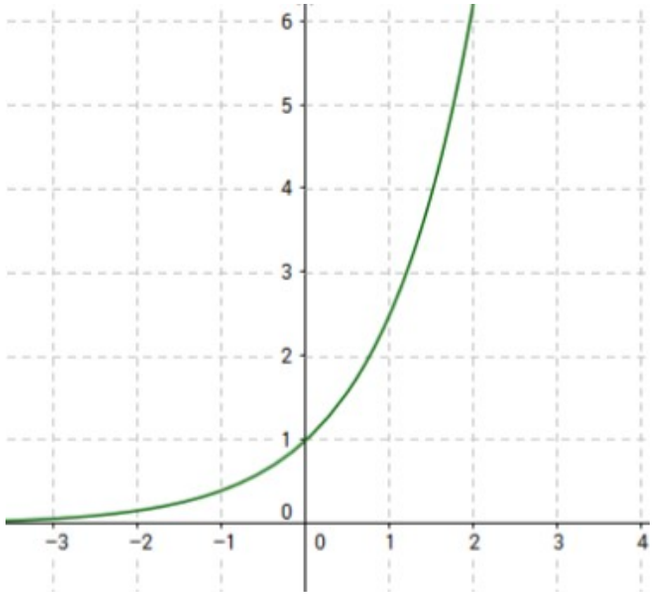
$y = 2.5^{-1} = \frac{1}{2.5} = .4$

$y = 2.5^0$

$y = 2.5^1$

$y = 2.5^2$

graph



growth

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Exponential growth: as x goes up, y goes up

Exponential decay: as x goes up, y goes down

There is an asymptote at the x-axis

(0, a) is the y-intercept

As long as a is positive:

b>1, exponential growth, b is the growth factor

0<b<1 (a fraction), exponential decay, b is the decay factor

y=0 is the asymptote and range is y>0

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Without graphing, determine whether the function represents exponential growth or exponential decay. Then find the *y*-intercept.

7. $y = 0.99\left(\frac{1}{3}\right)^x$

$a = .99 \quad b = \frac{1}{3}$

$b = \frac{1}{3} < 1$

decay

y-int: $(0, .99)$

9. $y = 185\left(\frac{5}{4}\right)^x$

$a = 185$

$b = \frac{5}{4} > 1$

growth

y-int: $(0, 185)$

11. $f(x) = 0.25(1.05)^x$

$a = .25$

$b = 1.05 > 1$

growth

y-int: $(0, .25)$

Creating equations for exponential growth and decay:

For growth or decay to be exponential, the amount must change by a fixed percentage each time period

$$A = a(1+r)^t$$

A is amount after t, time period.

a is the initial amount

r is the rate of growth or decay

t is the number of time periods

1 + r is the growth or decay factor

Ex: 45% → .45

% → change to a decimal

$b = 1 + r$

13. Suppose you deposit $1500 in a savings account that pays interest at an annual rate of 6%. No money is added or withdrawn from the account.

a. How much will be in the account after 5 years?

b. How much will be in the account after 20 years?

c. How many years will it take for the account to contain $2500?

d. How many years will it take for the account to contain $4000?

$A = a(1+r)^t$

$a = 1500$

$r = 6\% = .06$

$A = 1500(1+.06)^t = [A = 1500(1.06)^t]$

a) $t = 5$ $A = 1500(1.06)^5 \rightarrow 1500(1.338)$

$A = 2007.338 = \$2007.34$

b) $t = 20$ $A = 1500(1.06)^{20} = 1500(3.207) = 4810.703$

$\$4810.70$

c) $A = 2500$ $2500 = 1500(1.06)^t$

$2255.45 = 1500(1.06)^7$

$2534.22 = 1500(1.06)^9$

To get $2500 it will between 8 and 9 years

Write an exponential function to model each situation. Find each amount after the specified time.

a $r = -1.4\% = -.014$ $t = 18$

15. A population of 752,000 decreases 1.4% per year for 18 years.

$A = 752000(1 + -.014)^t$

$A = 752000(.986)^t$

$A = 752000(.986)^{18}$

$A = 583448.207$

$A = 583448$

For each annual rate of change, find the corresponding growth or decay factor.

17. + 45%

19. − 40%

21. + 28%

23. − 5%

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The growth or decay factor is 1+r.

If you do not have r, and they give you points, then $r = \frac{y_2 - y_1}{y_1}$

The order **does** matter.

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25. In 2009, there were 1570 (y_1) bears in a wildlife refuge. In 2010, the population had increased to approximately 1884 (y_2) bears. If this trend continues and the bear population is increasing exponentially, how many bears will there be in 2015?

$r = \frac{1884 - 1570}{1570} = .2$

$A = a(1+r)^t$

$a = 1570 \quad r = .2 \quad t = 5$

$A = 1570(1+.2)^5 \Rightarrow 1570(1.2)^5$

$A = 3906.66$

$A = 3907$ bears

$a = 4ft \rightarrow 4 \cdot 12 = 48in$

27. Your friend drops a rubber ball from 4 ft. You notice that its rebound is 32.5 in. on the first bounce and 22 in. on the second bounce.

a. What exponential function would be a good model for the height of the ball?

b. How high will the ball bounce on the fourth bounce?

y_1 y_2

$r = \frac{y_2 - y_1}{y}$

$r = \frac{22 - 32.5}{32.5}$

$r = -.323$

a) $A = 48(1 - .323)^t$

$A = 48(.677)^t$

b) $t = 4$ $A = 48(.677)^4$

$A = 10.08$ in

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a $r = -12\% = -.12$

29. A new truck that sells for $29,000 depreciates 12% each year. What is the value of the truck after 7 years?

$t = 7$

$A = 29000(1-.12)^7$

$A = 29000(.88)^7$

$A = 11851.59229$

$A = \$11851.59$ ←

31. The population of an endangered bird is decreasing at a rate of 0.75% per year. There are currently about 200,000 of these birds.

a. What exponential function would be a good model for the population of these endangered birds?

b. How many birds will there be in 100 years?

$r = -.75\%$

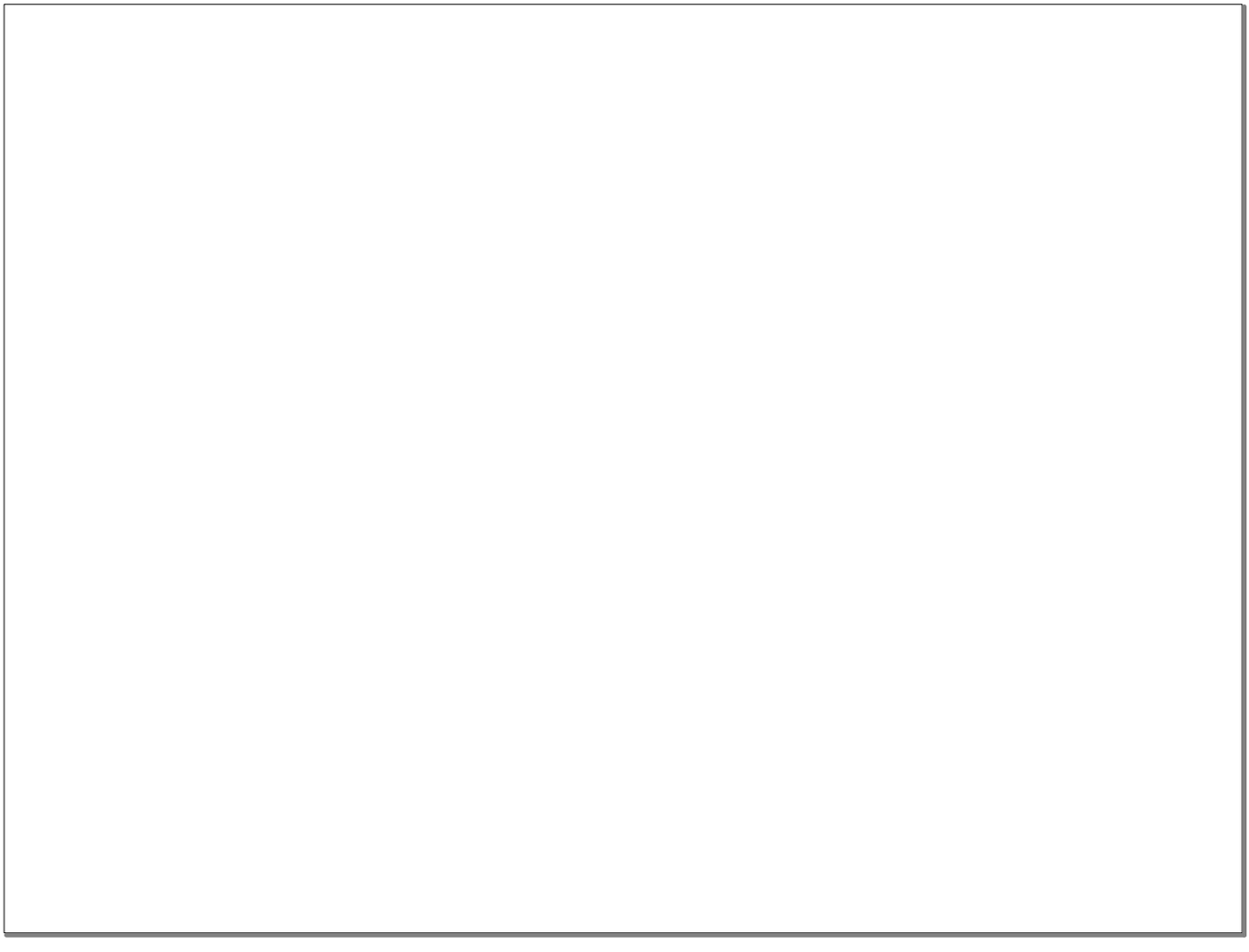
$r = -.0075$

$a = 200\,000$

a) $A = 200000(1-.0075)^t$

$A = 200000(.9925)^t$

b) $t = 100$ $\quad A = 200000(.9925)^{100}$

$A = 94206.645$

$A = 94207$ birds