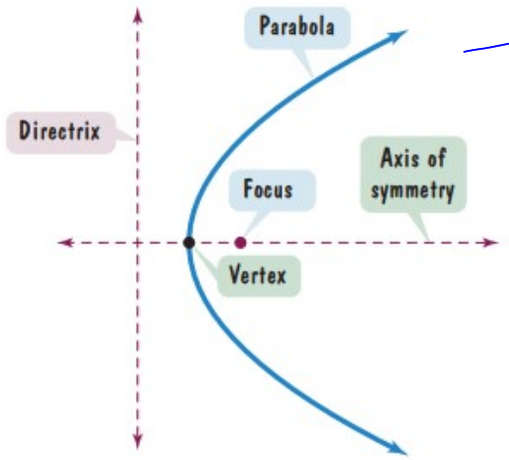
9.3 Parabola

Definition of a Parabola

A **parabola** is the set of all points in a plane that are equidistant from a fixed line, the **directrix**, and a fixed point, the **focus**, that is not on the line (see **Figure 9.29**).



$y^2 = x$

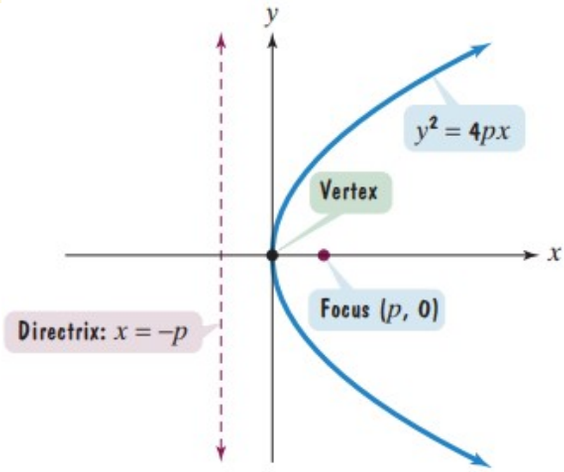
$y = \pm\sqrt{x}$

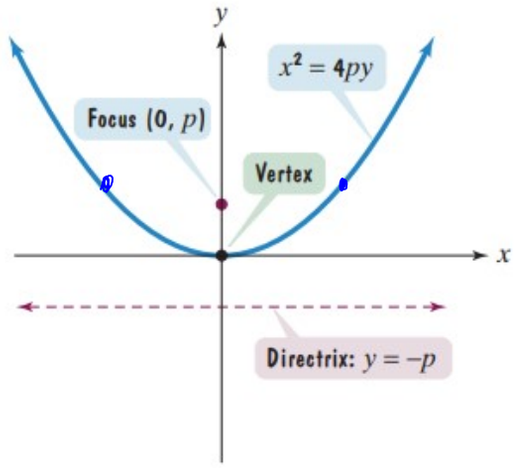
Standard Forms of the Equations of a Parabola

The **standard form of the equation of a parabola** with vertex at the origin is

$$y^2 = 4px \qquad \text{or} \qquad x^2 = 4py.$$

Figure 9.31(a) at the top of the next page illustrates that for the equation on the left, the focus is on the x-axis, which is the axis of symmetry. **Figure 9.31(b)** illustrates that for the equation on the right, the focus is on the y-axis, which is the axis of symmetry.





Apr 29-7:42 AM

EXAMPLE 1 Finding the Focus and Directrix of a Parabola

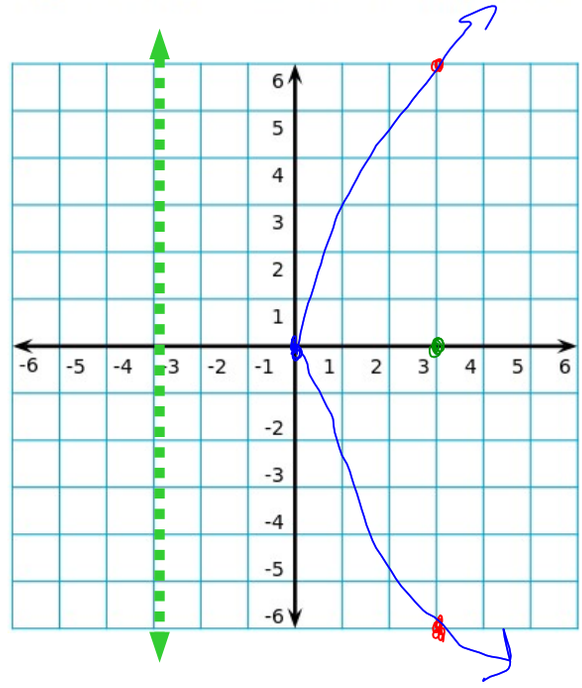
Find the focus and directrix of the parabola given by $y^2 = 12x$. Then graph the parabola.

horizontal

$y^2 = 4px$

$12 = 4p$

$p = 3$



Focus: $(3, 0)$

Directrix: $x = -3$

$y^2 = 12x$

$y^2 = 12(3) = 36$

$y = \pm 6 \rightarrow (3, 6)\ (3, -6)$

EXAMPLE 2 Finding the Focus and Directrix of a Parabola

Find the focus and directrix of the parabola given by $x^2 = -8y$. Then graph the parabola.

$-8 = 4p$

$p = -2$

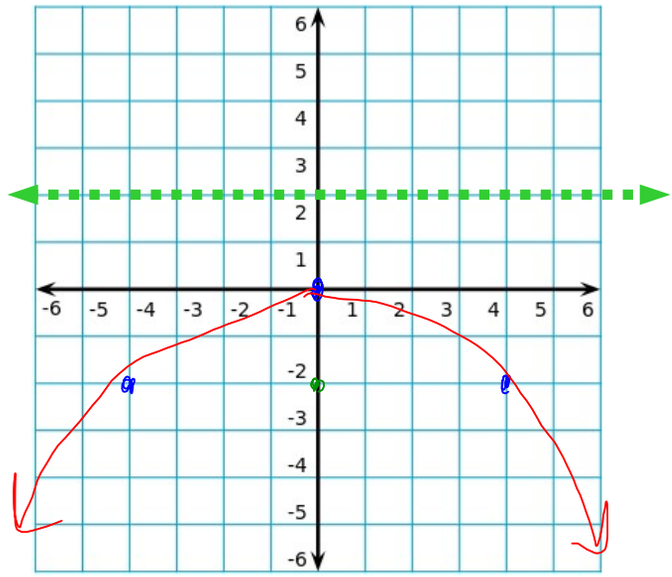
Directrix: $y = 2$

Focus: $(0, -2)$

$x^2 = -8(-2) = 16$

$x = \pm 4$

$(4, -2)$ $(-4, -2)$



Apr 29-7:45 AM

EXAMPLE 3 Finding the Equation of a Parabola from Its Focus and Directrix

Find the standard form of the equation of a parabola with focus $(5, 0)$ and directrix $x = -5$, shown in **Figure 9.35**.

Horizontal

opposite of directrix

$p = 5$

$y^2 = 4px$

$y^2 = 20x$

Horizontal		Vertical
$(y-k)^2 = 4p(x-h)$		$(x-h)^2 = 4p(y-k)$
(h, k)	Vertex	(h, k)
$(h+p, k)$	Focus	$(h, k+p)$
$x = h - p$	Directrix	$y = k - p$

Apr 29-7:47 AM

EXAMPLE 4 Graphing a Parabola with Vertex at (h, k)

Find the vertex, focus, and directrix of the parabola given by

$$(x - 3)^2 = 8(y + 1).$$

vertical

Then graph the parabola.

$h = 3$ $k = -1$ Vertex: $(3, -1)$

Directrix: $y = k - p$ $4p = 8$

$y = -1 - 2 = -3$ $p = 2$

Focus: $(h, k+p) = (3, -1+2) = (3, 1)$

Additional points: $(x-3)^2 = 8(y+1)$

$(x-3)^2 = 8(2)$

$(x-3)^2 = 16$

$x - 3 = \pm 4$

$x = 4 + 3 = 7$ $x = -4 + 3 = -1$

$(7, 1)$ and $(-1, 1)$

Apr 29-8:00 AM

EXAMPLE 5 Graphing a Parabola with Vertex at (h, k)

Find the vertex, focus, and directrix of the parabola given by

$$y^2 + 2y + 12x - 23 = 0.$$

Then graph the parabola.

$\left(\frac{2}{2}\right)^2 = (1)^2 = 1$

$y^2 + 2y + 1 = -12x + 23 + 1$

$(y+1)^2 = -12x + 24$

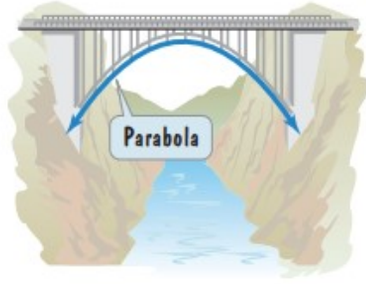
$(y+1)^2 = -12(x-2)$

Continue as usual

Apr 29-8:01 AM

Applications

Suspension bridge



Arch bridge

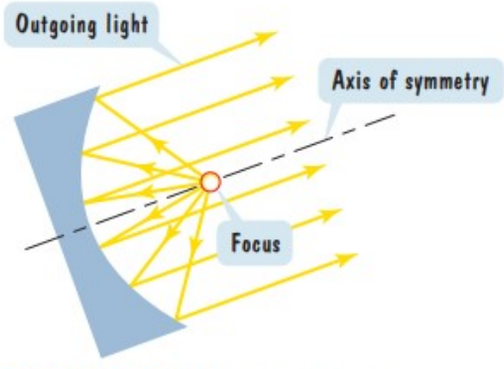


FIGURE 9.39(a) Parabolic surface reflecting light

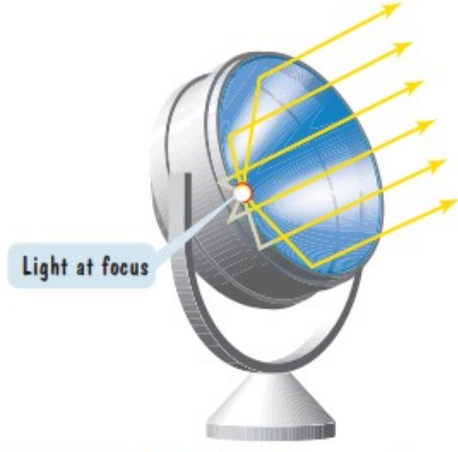


FIGURE 9.39(b) Light from the focus is reflected parallel to the axis of symmetry.

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Apr 30-11:20 AM