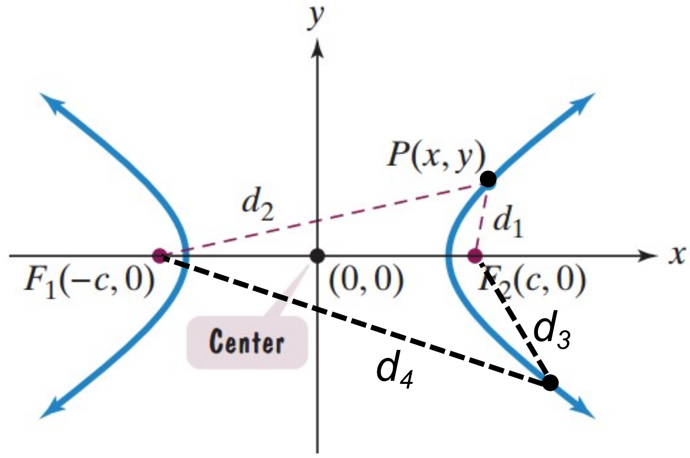
9.2 Hyperbolas

Definition: Set of all points whose distance from two central locations always subtracts to the same value.



$$d_1 - d_2 = d_3 - d_4$$

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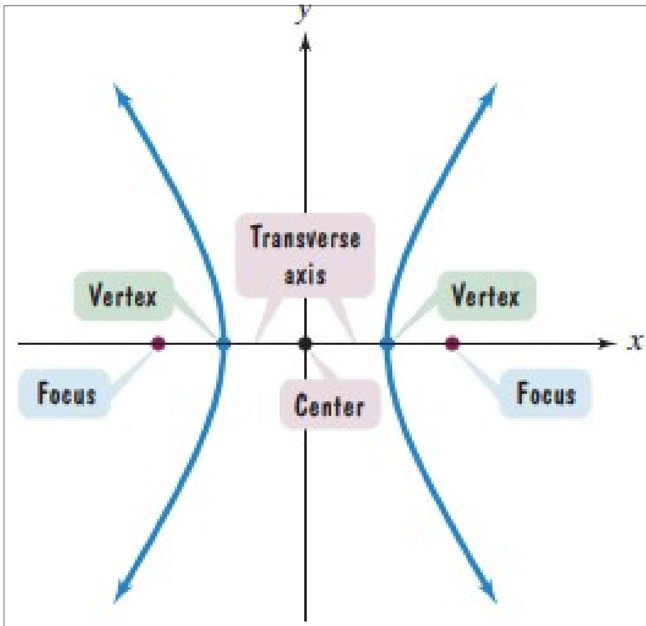

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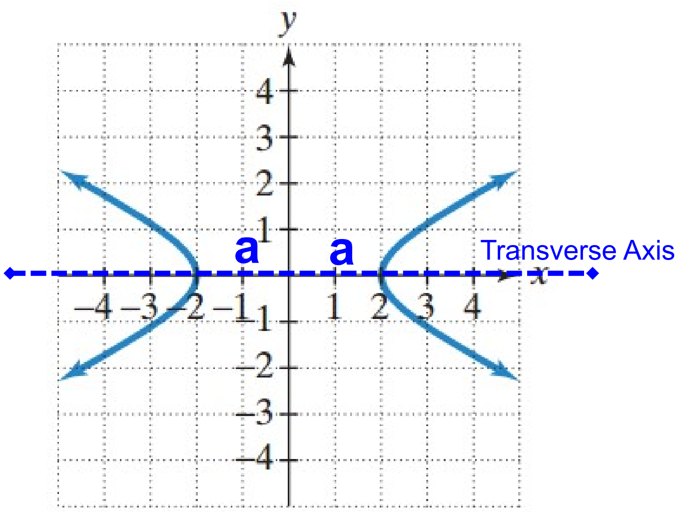
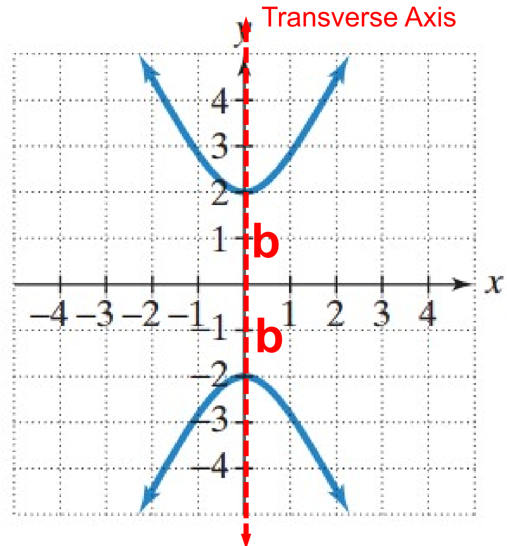
Types of Hyperbolas

Horizontal



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Vertical



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

Hyperbola:

equation is similar to th *ellipse* except it's **subtract** instead of **add**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

horizontal axis

$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$

vertical axis

differences between horizontal and vertical:

- transverse axis changes from **horiz** to **vert**
- in the equation:
  > x^2 and y^2 swap places

  and

  > a^2 and b^2 swap places
- The positive term tells you horizontal or vertical. a stays with x and b stays with y.

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For hyperbolas:

» x^2 and y^2 can trade places - vert/horiz

» if x^2 is positive (usually left) it's horizontal

» if y^2 is positive (usually left) it's vertical

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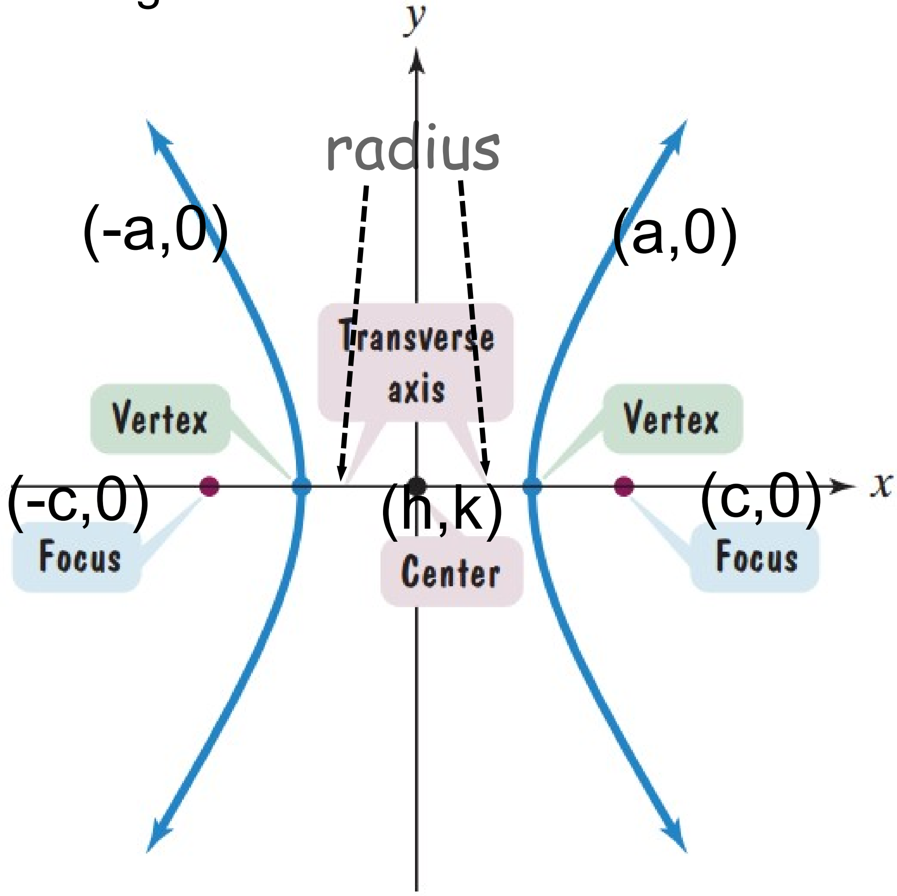
For hyperbola, the the relationship between *a,b, & c* changes. We now use: $\mathbf{c^2 = a^2 + b^2}$



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EXAMPLE 1 Finding Vertices and Foci from a Hyperbola's Equation

Find the vertices and locate the foci for each of the following hyperbolas with the given equation:

a. $\frac{x^2}{16} - \frac{y^2}{9} = 1$ **b.** $\frac{y^2}{9} - \frac{x^2}{16} = 1.$

a. horizontal

vertices: $(\pm a, 0)$

$a^2 = 16$ $a = 4$

vertices: $(4,0)$ $(-4,0)$

$c^2 = a^2 + b^2$

$c^2 = 16 + 9 = 25$

$c = 5$

foci: $(5,0)$ $(-5,0)$

b. vertical

vertices: $(0, \pm b)$

$b^2 = 9$ $b = 3$

verties: $(0,3)$ $(0,-3)$

Foci: $(0,5)$ $(0,-5)$

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EXAMPLE 2 Finding the Equation of a Hyperbola from Its Foci and Vertices

Find the standard form of the equation of a hyperbola with foci at $(0, -3)$ and $(0, 3)$ and vertices $(0, -2)$ and $(0, 2)$, shown in **Figure 9.19**.

vertical $b = 2$ $b^2 = 4$

$c = 3$ $c^2 = 9$

$c^2 = a^2 + b^2$ $9 = a^2 + 4$

$a^2 = 5$

$$\frac{y^2}{4} - \frac{x^2}{5} = 1$$

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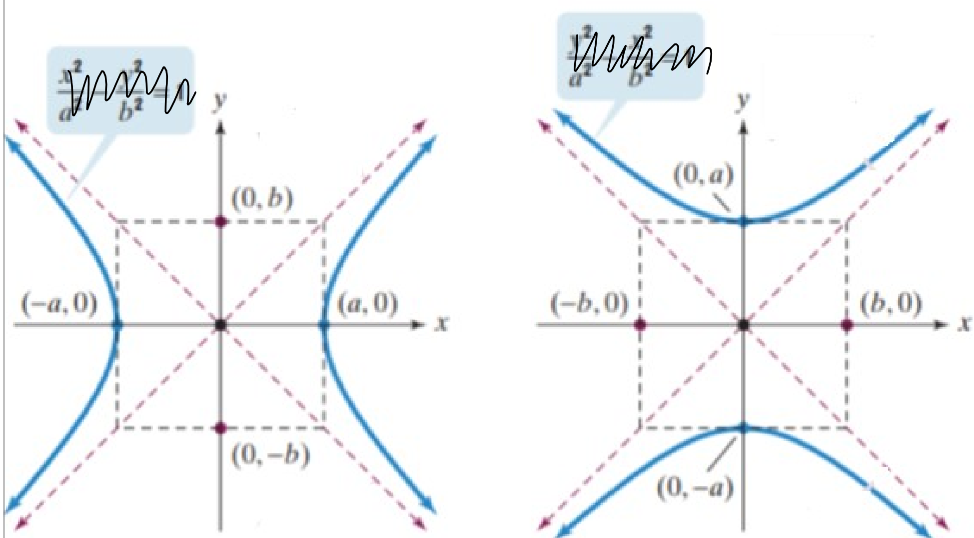
Asymptotes of a Hyperbola

The asymptotes pass through the center of the hyperbola



Slope of the asymptotes:

$$y=\pm\frac{b}{a}$$

Graphing Hyperbolas

1. Determine and graph the vertices
2. Determine and graph the asymptotes
3. Draw the two branches going through the vertices and approaching the asymptotes

EXAMPLE 3 Graphing a Hyperbola

slope

Graph and locate the foci: $\frac{x^2}{25} - \frac{y^2}{16} = 1$. What are the equations of the asymptotes?

horizontal

$a^2 = 25$

vertices: $(\pm 5, 0)$

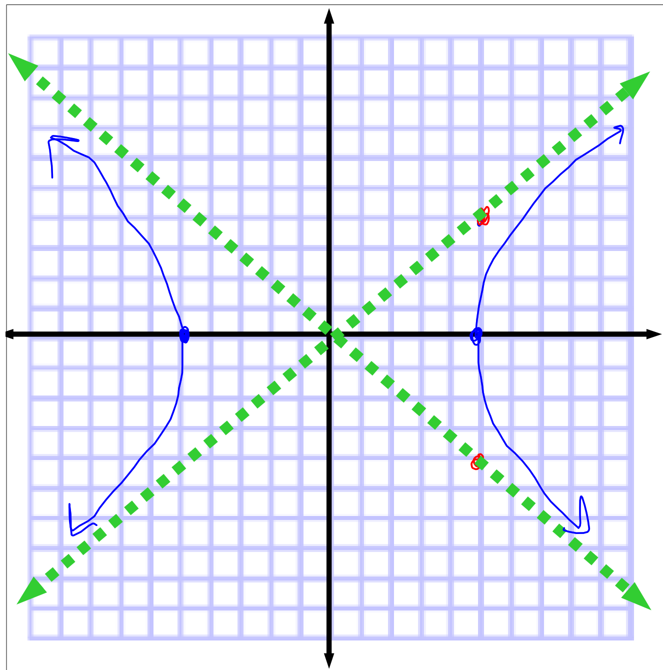
$a = 5$

$c^2 = 25 + 16 = 41$ $c = \sqrt{41}$

Foci: $(\pm \sqrt{41}, 0)$

$a = 5$ $b = 4$

slope of Asymptotes: $\pm \frac{4}{5}$



EXAMPLE 4 Graphing a Hyperbola

Graph and locate the foci: $9y^2 - 4x^2 = 36$. What are the equations of the asymptotes?

36 36 36

We are not going to graph this one, but how would you put it in standard form to be able to graph it?

$$\frac{y^2}{4} - \frac{x^2}{9} = 1$$

vertical

b/c y^2 is positive

$b^2 = 4$

$b = 2$

$a^2 = 9$

$a = 3$

Slope of Asymptote:

$\pm \frac{2}{3}$

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Horizontal

Center: (h,k)

Vertical

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad\qquad \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

$$c^2 = a^2 + b^2 \qquad\qquad c^2 = a^2 + b^2$$

$(h \pm a, k)$ ⟵ Vertices ⟶ $(h, k \pm b)$

Slopes

$y = \pm\frac{b}{a}$ ⟵ Asymptotes ⟶ $y = \pm\frac{b}{a}$

$(h \pm c, k)$ ⟵ Foci ⟶ $(h, k \pm c)$

EXAMPLE 5 Graphing a Hyperbola Centered at (h, k)

Graph: $\frac{(x-2)^2}{16} - \frac{(y-3)^2}{9} = 1$. Where are the foci located? What are the equations of the asymptotes? horizontal

Center: $(2,3)$

$a^2 = 16$ $b^2 = 9$

Vertices: $(2 \pm 4, 3)$

$a = 4$ $b = 3$

$(6,3)$ $(-2,3)$

$c^2 = 16 + 9 = 25$

$c = 5$

Foci: $(2 \pm 5, 3)$

$(7,3)$ $(-3,3)$

Slopes of Asymptotes: $\pm \frac{3}{4}$

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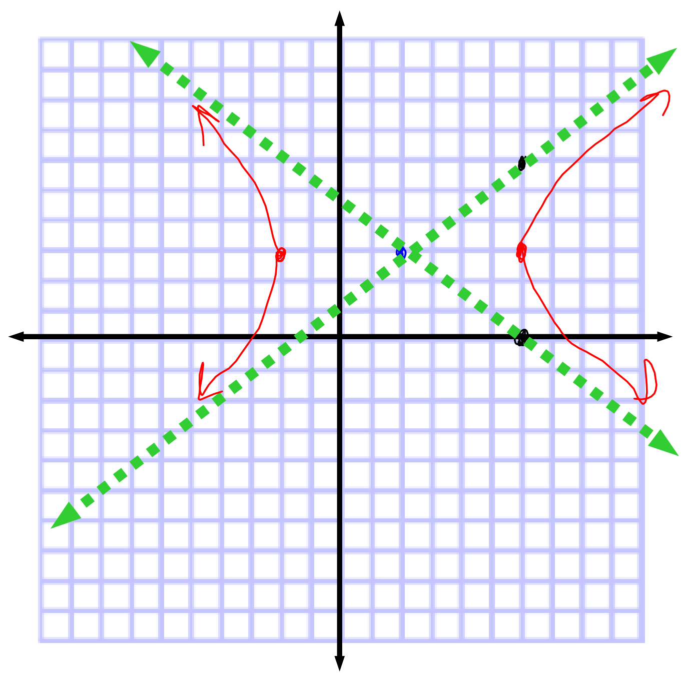

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Check Point 5 Graph: $\frac{(x-3)^2}{4} - \frac{(y-1)^2}{1} = 1$. Where are the foci located? What are the equations of the asymptotes?

Center: $(3, 1)$ $a = 2$ $b = 1$

vertices: $(5,1)$ $(1,1)$ $c^2 = 5$

$c = \sqrt{5}$

Foci: $(3 \pm \sqrt{5}, 1)$

Slope of A: $\pm \frac{1}{2}$

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EXAMPLE 6 Graphing a Hyperbola Centered at (h, k)

Graph: $4x^2 - 24x - 25y^2 + 250y - 489 = 0$. Where are the foci located? What are the equations of the asymptotes?

$$4x^2 - 24x - 25y^2 + 250y = 489$$

$$4(x^2 - 6x) - 25(y^2 - 10y) = 489$$

$$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9 \qquad \left(\frac{-10}{2}\right)^2 = (-5)^2 = 25$$

$$4(x^2 - 6x + 9) - 25(y^2 - 10y + 25) = 489 + 36 - 625$$

$$\frac{4(x-3)^2}{-100} - \frac{25(y-5)^2}{-100} = \frac{-100}{-100}$$

$$-\frac{(x-3)^2}{25} + \frac{(y-5)^2}{4} = 1$$

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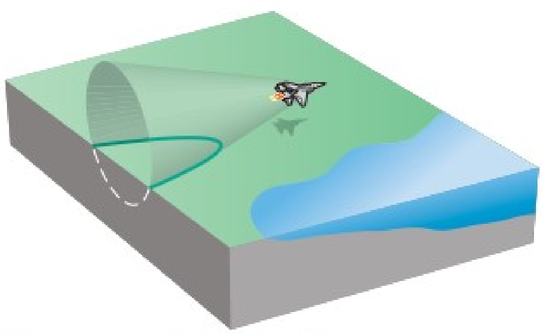
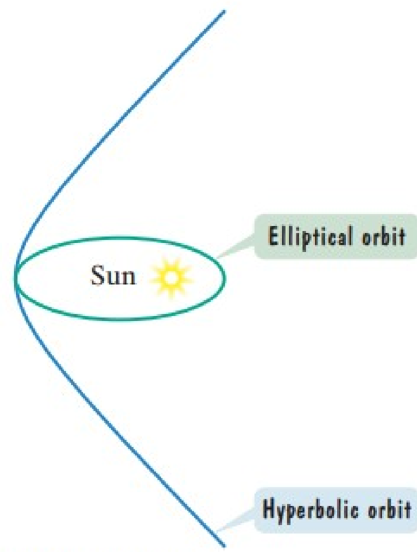
Write $4x^2 - 24x - 9y^2 - 90y - 153 = 0.$ in standard hyperbolic form.

Applications



The hyperbolic shape of a sonic boom



Orbits of comets

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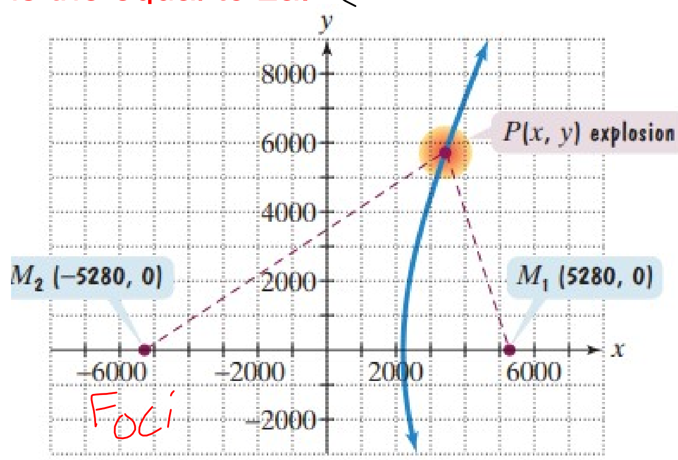
EXAMPLE 7 An Application Involving Hyperbolas

An explosion is recorded by two microphones that are 2 miles apart. Microphone M_1 received the sound 4 seconds before microphone M_2. Assuming sound travels at 1100 feet per second, determine the possible locations of the explosion relative to the location of the microphones.

Create a hyperbolic function to model the situation.

Note: In developing the standard form of the hyperbola, they found that the difference of the distances from the foci is the equal to 2a.



HW

Pg. 945 #1-6, 14-24 even, 34-46 even,