9.1 The Ellipse

Apr 13-7:19 AM

Mathematics is present in the movements of planets, bridge and tunnel construction, navigational systems used to keep track of a ship's location, manufacture of lenses for telescopes, and even in a procedure for disintegrating kidney stones. The mathematics behind these applications involves conic sections. Conic sections are curves that result from the intersection of a right circular cone and a plane. **Figure 9.1** illustrates the four conic sections: the circle, the ellipse, the parabola, and the hyperbola.

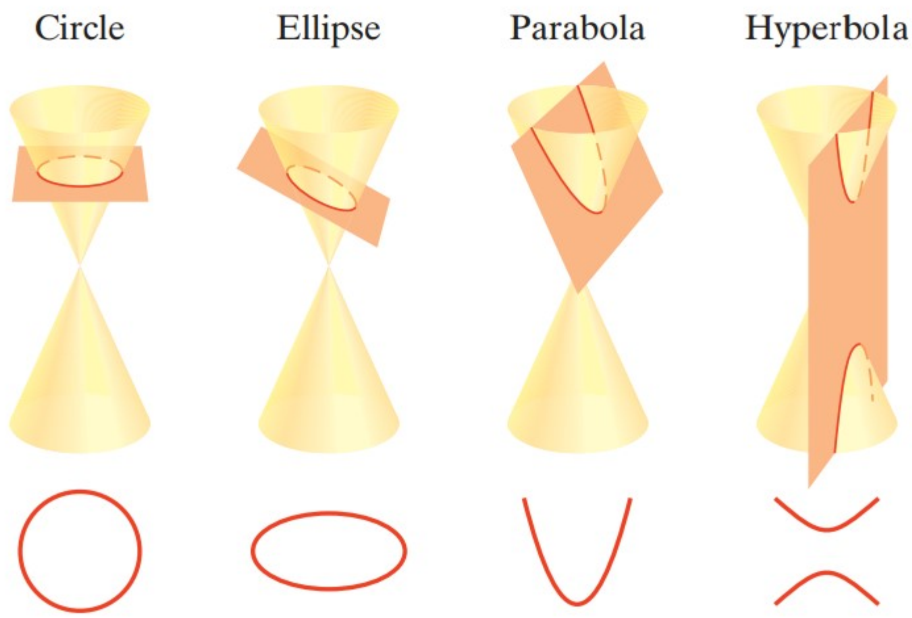


FIGURE 9.1 Obtaining the conic sections by intersecting a plane and a cone

Apr 13-8:25 AM

We have investigated graphs of circles and their equations.

Similar to these equations are those of **the ellipse**.

Standard Form of an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 > b^2$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Apr 13-7:43 AM

Vocab **you** need to know:

- focus (plural is foci)
- center
- vertex (plural is vertices) point of intersection on major axis
- major axis
- minor axis

Definition of an Ellipse

An **ellipse** is the set of all points, P, in a plane the sum of whose distances from two fixed points, F_1 and F_2, is constant (see **Figure 9.3**). These two fixed points are called the **foci** (plural of **focus**). The midpoint of the segment connecting the foci is the **center** of the ellipse.

***x*-intercepts: Set $y = 0$.**

$$\frac{x^2}{a^2} = 1$$
$$x^2 = a^2$$
$$x = \pm a$$

***y*-intercepts: Set $x = 0$.**

$$\frac{y^2}{b^2} = 1$$
$$y^2 = b^2$$
$$y = \pm b$$

$d_1 + d_2 = d_3 + d_4$

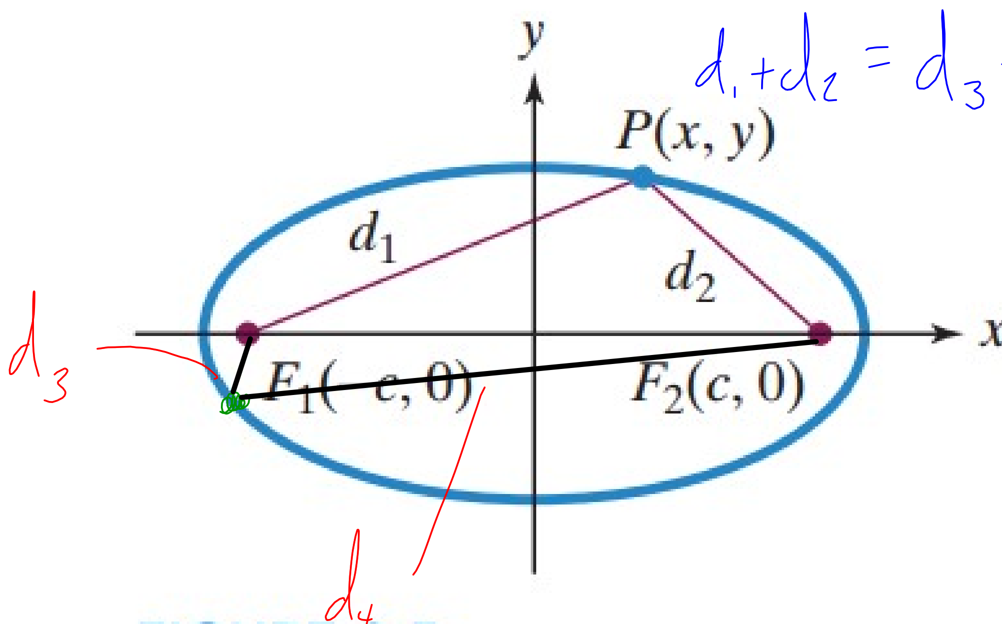


FIGURE 9.5

Major axis = longer one of the two axes

Vertex = one at each end of the major axis (it's on the line)

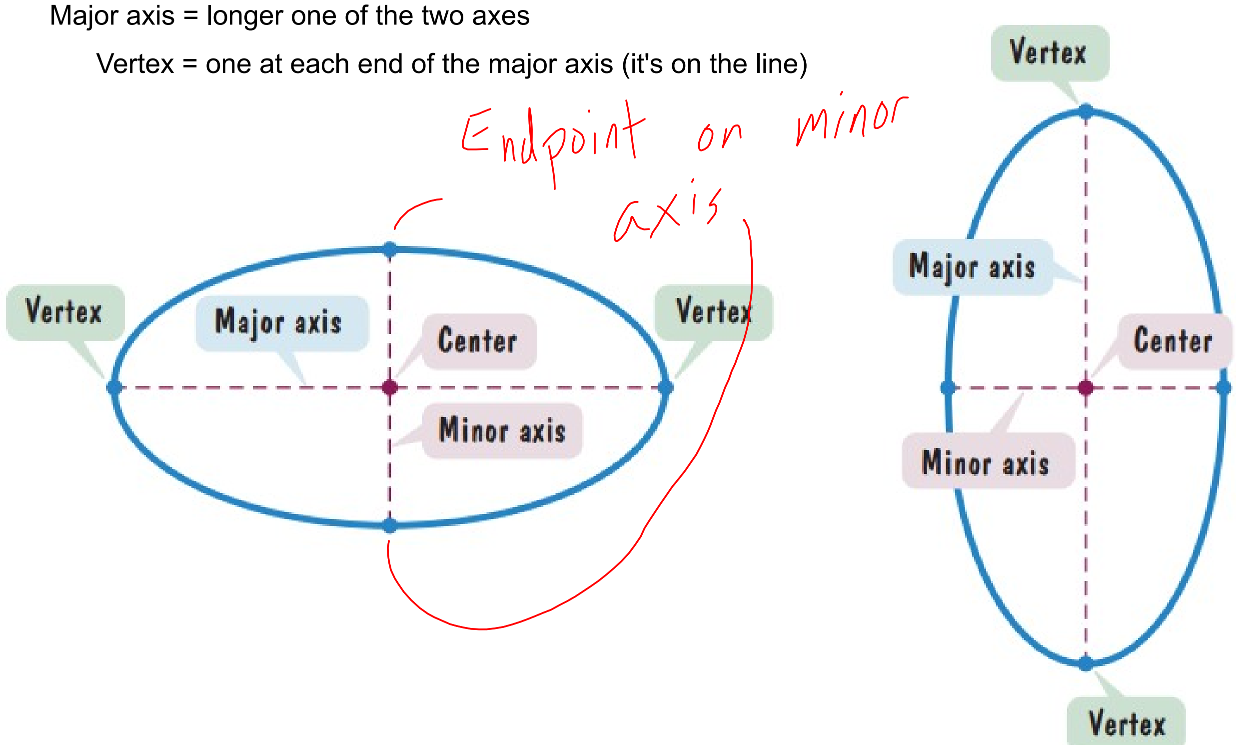


FIGURE 9.4 Horizontal and vertical elongations of an ellipse

Apr 13-8:34 AM

Figure 9.6 illustrates that the vertices are on the major axis, a units from the center. The foci are on the major axis, c units from the center. For both equations, $b^2 = a^2 - c^2$. Equivalently, $\underline{c^2 = a^2 - b^2}$. $\longrightarrow$ Foci $a^2 > b^2$

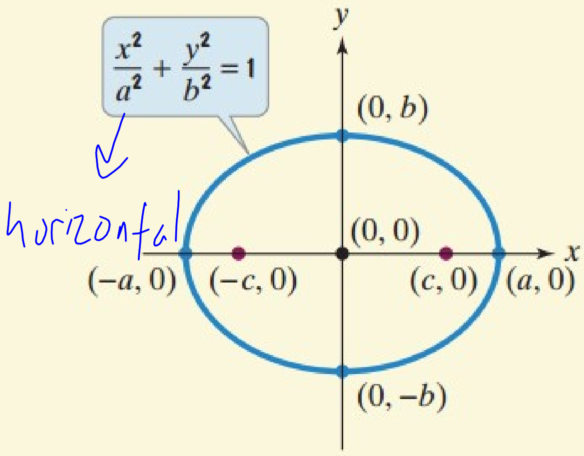


FIGURE 9.6(a) Major axis is horizontal with length $2a$.

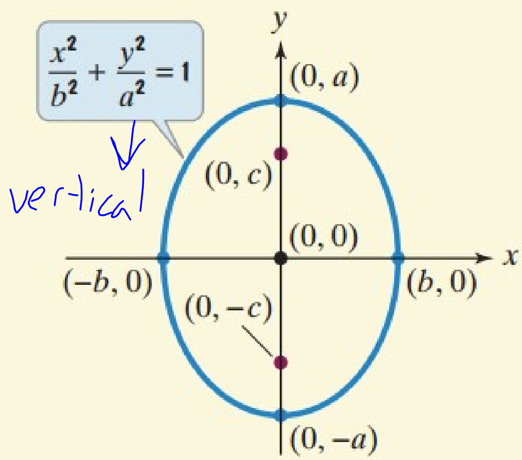


FIGURE 9.6(b) Major axis is vertical with length $2a$.

Question: How do you know when $\mathbf{a^2}$ is on the <u>left</u> or the <u>right</u>?

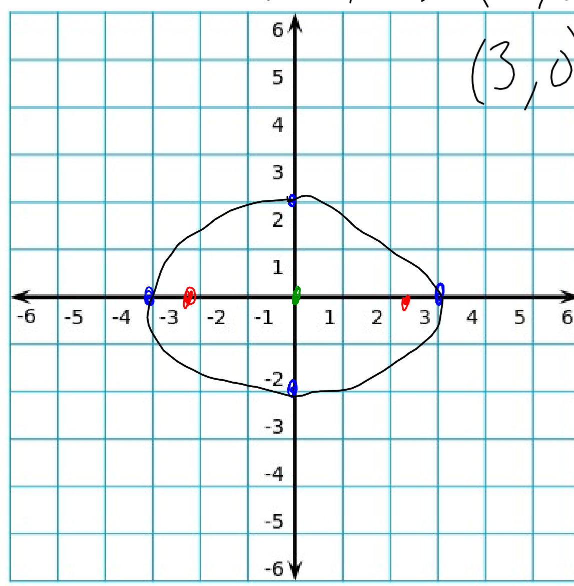
Answer: $\mathbf{a^2}$ is always the larger denominator

EXAMPLE 1 Graphing an Ellipse Centered at the Origin

Graph and locate the foci: $\frac{x^2}{9} + \frac{y^2}{4} = 1.$

vertices: $(-3,0)$ $(3,0)$

horizontal: Bigger # under x.



$a^2 = 9$ $b^2 = 4$

x-int: ± 3 y-int: ± 2

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \pm\sqrt{5} = \pm 2.24$

Center: $(0,0)$

EXAMPLE 2 Graphing an Ellipse Centered at the Origin

Graph and locate the foci: $\frac{25x^2}{400} + \frac{16y^2}{400} = \frac{400.}{400}$ → must $= 1$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$ vertical