5.5

Solving Trigonometric Equations

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Steps to solving trig equations:

- Isolate the trigonometric function (treated just like a variable)

- Find the solutions on the unit circle that corresponds to the equation

- Since trig functions are periodic, you must add $2\pi n$ as a means to find all possible solutions.

sin, cos

tan → add πn

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}$ and $\frac{5\pi}{3}$

$x = \frac{\pi}{3} + 2\pi n$ $\quad$ $x = \frac{5\pi}{3} + 2\pi n$

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Solve the equation: $3 \sin x - 2 = 5 \sin x - 1$.

$u = \sin x$

$3u - 2 = 5u - 1$

$-3u + 1 \quad -3u + 1$

$-1 = 2u$

$u = -1/2$

$\sin x = -1/2$

$x = \frac{7\pi}{6} + 2\pi n$ or $x = \frac{11\pi}{6} + 2\pi n$

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On your own: Solve the equation: $5 \sin x = 3 \sin x + \sqrt{3}$.

$-3\sin x \quad -3\sin x$

$$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$$

$$\sin x = \frac{\sqrt{3}}{2}$$

$x = \pi/3 + 2\pi n \qquad x = 2\pi/3 + 2\pi n$

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Solve the equation: $\tan 3x = 1, \quad 0 \le x < 2\pi.$

Not looking for all solutions

Let's replace 3x with u for a moment.

Check the interval at the end.

$\tan = \frac{\sin}{\cos} = \frac{y}{x}$

$u = 3x$

$\tan u = 1$

$u = \frac{\pi}{4} + \pi n$

$\frac{1}{3} \cdot 3x = \left(\frac{\pi}{4} + \pi n\right)\frac{1}{3}$

$x = \frac{\pi}{12} + \frac{\pi n}{3} = \frac{\pi}{12} + \frac{4\pi n}{12}$

$n=0$	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$
$x=\frac{\pi}{12}$	$x=\frac{5\pi}{12}$	$x=\frac{3\pi}{4}$	$x=\frac{13\pi}{12}$	$x=\frac{17\pi}{12}$	$x=\frac{7\pi}{4}$

On your own Solve the equation: $\tan 2x = \sqrt{3}, 0 \le x < 2\pi$.

$\tan 2x = \sqrt{3}$

$\frac{1}{2} \cdot 2x = \left(\frac{\pi}{3} + \pi n\right)\frac{1}{2}$

$x = \frac{\pi}{6} + \frac{\pi n}{2} = \frac{\pi}{6} + \frac{3\pi n}{6}$

$n = 0$ $n = 1$ $n = 2$ $n = 3$

$\left(x = \frac{\pi}{6}\right.$ $x = \frac{2\pi}{3}$ $x = \frac{7\pi}{6}$ $\left.x = \frac{5\pi}{3}\right)$

Solve the equation: $\sin\frac{x}{2} = \frac{\sqrt{3}}{2}, 0 \le x < 2\pi.$

Again, substitute u in temporarily and check the interval at the end.

$u = \frac{x}{2}$ $\sin u = \frac{\sqrt{3}}{2}$

$u = \frac{\pi}{3} + 2\pi n$ $u = \frac{2\pi}{3} + 2\pi n$

$2 \cdot \frac{x}{2} = \left(\frac{\pi}{3} + 2\pi n\right)$ $\frac{x}{2} = \left(\frac{2\pi}{3} + 2\pi n\right) 2$

$x = \frac{2\pi}{3} + 4\pi n$ $x = \frac{4\pi}{3} + 4\pi n$

$n = 0$ $n = 0$

$x = \frac{2\pi}{3}$ $x = \frac{4\pi}{3}$

Solve the equation: $2\cos^2 x + \cos x - 1 = 0, \quad 0 \le x < 2\pi.$

What could we plug in temporarily to make this problem easier to deal with?

$u = \cos x$

$2u^2 + u - 1 = 0$

$a = 2 \quad b = 1 \quad c = -1$

$a \cdot c$	b
factors	add $\div a$

-2	1
$2 \cdot -1$	$\frac{2}{2} - \frac{1}{2} = 1$ ✓

$\frac{1}{1} \quad \frac{-1}{2}$

on bottom goes in front

on top goes behind

$(1u + 1)(2u - 1) = 0$

$u + 1 = 0 \qquad 2u - 1 = 0$

$u = -1 \qquad u = \frac{1}{2}$

$\cos x = -1 \qquad \cos x = \frac{1}{2}$

$x = \pi + 2\pi n \qquad x = \frac{\pi}{3} + 2\pi n \qquad x = \frac{5\pi}{3} + 2\pi n$

$x = \pi \qquad x = \frac{\pi}{3} \qquad x = \frac{5\pi}{3}$

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On your own

Solve the equation: $2\sin^2 x - 3\sin x + 1 = 0, \quad 0 \le x < 2\pi.$

$u = \sin x$

$2u^2 - 3u + 1 = 0$

2	-3
$1 \cdot 2$	$1 + 2 = 3$
$-1 \cdot -2$	$-\frac{1}{2} - \frac{2}{2} = -3$
$-\frac{1}{2}$	$-\frac{1}{1}$

$(2u-1)(u-1) = 0$

$u = \frac{1}{2}$ $\quad$ $u = 1$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = 1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{\pi}{2}$

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Solve the equation: $4\sin^2 x - 1 = 0, \quad 0 \le x < 2\pi.$

$u = \sin x$

$4u^2 - 1 = 0$

$+1 \quad +1$

$\frac{4u^2}{4} = \frac{1}{4}$

$\sqrt{u^2} = \pm\sqrt{\frac{1}{4}}$

$u = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$u = \frac{1}{2}$ $\quad$ $u = -\frac{1}{2}$

$\sin x = \frac{1}{2}$ $\quad$ $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

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Solve the equation: $\tan x \sin^2 x = 3 \tan x, \quad 0 \le x < 2\pi.$

$x = \tan x$

$y = \sin x$

$$xy^2 = 3x$$
$$-3x \qquad -3x$$

$$xy^2 - 3x = 0$$
$$x(y^2 - 3) = 0$$

$$\tan x \sin^2 x = 3\tan x$$
$$-3\tan x \qquad -3\tan x$$

$$\tan x \sin^2 x - 3\tan x = 0$$
$$\tan x(\sin^2 x - 3) = 0$$

$\tan x = 0$

$x = 0 + \pi n$

$n = 0$, $x = 0$

$n = 1$, $x = \pi$

$n = 2$, $x = 2\pi$ out of interval

$$\sin^2 x - 3 = 0$$
$$+3 \qquad +3$$
$$\sqrt{\sin^2 x} = \pm\sqrt{3}$$
$$\sin x \neq \pm\sqrt{3} = \pm 1.73$$

$\sqrt{3}$ is outside the range of $\sin x$

$x = 0, \pi$

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Solve the equation: $\cos 2x + 3\sin x - 2 = 0, \quad 0 \le x < 2\pi.$

5.3 $\cos 2x = 1 - 2\sin^2 x$ (double ∠)

$1 - 2\sin^2 x + 3\sin x - 2 = 0$

$$\frac{-2\sin^2 x + 3\sin x - 1}{-1} = \frac{0}{-1}$$

$2\sin^2 x - 3\sin x + 1 = 0$

2	-3
-1 · -2	$-\frac{1}{2}$ $-\frac{2}{2}$

$\rightarrow -\frac{1}{2} \quad -\frac{1}{1}$

$(2\sin x - 1)(\sin x - 1) = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = 1$

$x = \pi/6, 5\pi/6$ $\qquad$ $x = \pi/2$

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On your own

Solve the equation: $\cos 2x + \sin x = 0, \quad 0 \le x < 2\pi.$

$$\frac{1 - 2\sin^2 x + \sin x}{-1} = \frac{0}{-1}$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$\begin{array}{c|c} -2 & -1 \\ \hline -2 \cdot 1 & -\frac{2}{2} + \frac{1}{2} = -1 \end{array} \rightarrow \frac{-1}{1} \quad \frac{1}{2}$$

$$(\sin x - 1)(2\sin x + 1) = 0$$

$\sin x = 1$ $\qquad$ $\sin x = -\frac{1}{2}$

$x = \frac{\pi}{2}$ $\qquad$ $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

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Solve each equation, correct to ~~four~~ 2 decimal places, for $0 \le x < 2\pi$:

a. $\tan x = 12.8044$ **b.** $\cos x = -0.4317$.

radians

a) $\tan^{-1}(\overset{y}{12.8044}) = x$

1.49 radians

b) $\cos^{-1}(-.4317) = 2.01$ radians $= x$

Solve the equation, correct to four decimal places, for $0 \le x < 2\pi$:

$$\sin^2 x - \sin x - 1 = 0.$$

$-1 \mid -1$ not factorable

$$\sin x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$\sin x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$\sin x = \frac{1+\sqrt{5}}{2} \qquad \sin x = \frac{1-\sqrt{5}}{2}$$

$\sin^{-1}((1+\sqrt{5}) \div 2) =$ Error Outside the range

$\sin^{-1}((1-\sqrt{5}) \div 2) = -.6662$

$x = -.6662 < 0$, outside the interval

$x = -.6662 + 2\pi n$

$n = 0$ $\quad$ $n = 1$

$x = -.6662$ $\quad$ $x = 5.617$

$x = 5.617$

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On your own:

Solve the equation, correct to four decimal places, for $0 \leq x < 2\pi$:

$$\cos^2 x + 5\cos x + 3 = 0.$$

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HW

Pg. 674 # 11-37 odd, 55, 57, 85-91 odd, 131, 133

You must show work!