Lesson 2.4
Dividing Polynomials
and
The Remainder Theorem

Oct 13-9:30 AM

Use long division to divide

$6497 \div 23$

$$\begin{array}{r} 282 \\ 23\overline{)6497} \\ -46\downarrow \\ \hline 189 \\ -184\downarrow \\ \hline 57 \\ -46 \\ \hline 11 \end{array}$$

$282\ R\ 11$

$282\ \frac{11}{23}$

Divide $x^2 + 10x + 21$ by $x + 3$.

$$x+3 \overline{)\, x^2 + 10x + 21}$$

$x + 7$

$-(x^2 + 3x)$ ↓

$7x + 21$

$-(7x + 21)$

0

$x + 7$

$x(x+3)$

$x^2 + 3x$

$7(x+3)$

$7x + 21$

Note:

When doing long division with polynomials:

If there is no remainder, the divisor is a factor of the polynomial.

If there is a remainder, the divisor is NOT a factor.

This is known as the **Factor Theorem**

Divide using long division:

$$(6x^5 + 9x^4 - 4x^2 - 6x) \div (3x^3 - 2)$$

$$\begin{array}{r} 2x^2 + 3x \\ 3x^3 - 2 \overline{)\, 6x^5 + 9x^4 + 0x^3 - 4x^2 - 6x + 0} \\ \underline{-(6x^5 + 0x^4 + 0x^3 - 4x^2)} \\ 9x^4 + 0x^3 + 0x^2 - 6x \\ \underline{-(9x^4 + 0x^3 + 0x^2 - 6x)} \\ 0 \quad 0 \end{array}$$

$2x^2 + 3x$

b/c remainder is 0

$3x^3 - 2$ and $2x^2 + 3x$ are factors

$2x^2(3x^3 - 2) = 6x^5 - 4x^2$

$3x(3x^3 - 2) = 9x^4 - 6x$

Sep 28-9:49 AM

Divide $4 - 5x - x^2 + 6x^3$ by $3x - 2$.

$$2x^2 + x - 1$$
$$3x-2 \overline{)\,6x^3 - x^2 - 5x + 4}$$
$$-(6x^3 - 4x^2)$$
$$3x^2 - 5x$$
$$-(3x^2 - 2x)$$
$$-3x + 4$$
$$-(-3x + 2)$$
$$2$$

$2x^2(3x-2)$
$6x^3 - 4x^2$
$x(3x-2)$
$3x^2 - 2x$

$$2x^2 + x - 1 + \frac{2}{3x-2}$$

Sep 20-8:47 PM

We can use **synthetic division** to divide polynomials if the divisor is of the form $x - c$.

You can only use synthetic division when the divisor is linear, and the coefficient is 1.

Notice the notation. Due to the negative sign, we will have to change the sign of the number.

With synthetic division, you are only using the coefficients, but make sure to include any missing terms.

c | coefficients and constant

Use synthetic division to divide $5x^3 + 6x + 8$ by $x + 2$.

$0x^2$

$c = -2$

-2	5	0	6	8
	↓	-10	20	-52
	5	-10	26	-44
	x^2	x	Constant	R

$$5x^2 - 10x + 26 - \frac{44}{x+2}$$

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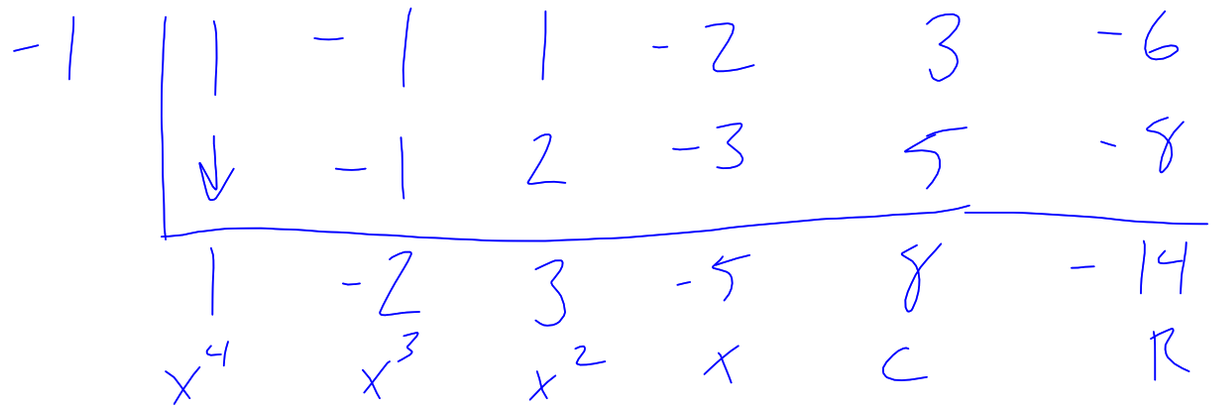
Try it on your own:

Divide using synthetic division:

$(x^5 - x^4 + x^3 - 2x^2 + 3x - 6) \div (x + 1)$



$x^4 - 2x^3 + 3x^2 - 5x + 8 - \frac{14}{x+1}$

The Remainder Theorem

If the polynomial $f(x)$ is divided by $x - c$, then the remainder is $f(c)$.

Given $f(x) = x^3 - 4x^2 + 5x + 3$, determine if there is a remainder with a divisor of x + 2

$c = -2$

$f(-2) = (-2)^3 - 4(-2)^2 + 5(-2) + 3$

$-8 - 16 - 10 + 3 = -31$

↑ remainder

$x+2$ is not a factor

Given $f(x) = x^3 - 4x^2 + 5x + 3$, find the remainder when c = 2 → root

factor: $x-2$

$f(2) = (2)^3 - 4(2)^2 + 5(2) + 3$

$8 - 16 + 10 + 3 = 5$

$x-2$ is not a factor

Use Remainder Theorem to find f(¼):

$$f(x) = 8x^4 + 6x^3 + 2x^2 + 4x + 10$$

$$8\left(\frac{1}{4}\right)^4 + 6\left(\frac{1}{4}\right)^3 + 2\left(\frac{1}{4}\right)^2 + 4\left(\frac{1}{4}\right) + 10$$

$$8\left(\frac{1}{256}\right) + 6\left(\frac{1}{64}\right) + 2\left(\frac{1}{16}\right) + 1 + 10$$

$$\frac{8}{256} + \frac{6}{64} + \frac{2}{16} + 1 + 10$$

$$\frac{1}{32} + \frac{3}{32} + \frac{4}{32} + 1 + 10$$

$$11\frac{8}{32} = 11\frac{1}{4} = \frac{45}{4}$$

Find all the solutions (zeros) of the function, given that -½ is a root.

$$f(x) = 12x^3 + x^2 - \frac{9}{2}x - 1$$

$x = -\frac{1}{2}$ factor: $(x + \frac{1}{2})$

possible zeros: $\pm \frac{\text{factors of the constant}}{\text{factors of the LC}}$

possible zeros: $\pm \frac{1}{1, 2, 3, 4, 6, 12}$

$-\frac{1}{2}$	12	1	$-\frac{9}{2}$	-1
	↓	-6	$\frac{5}{2}$	1
	12	-5	$-\frac{4}{2}$	0

-2

$(x + \frac{1}{2})(12x^2 - 5x - 2) = 0$

$-24 \mid -5$

$-8 \cdot 3 \mid -8 + 3 = -5$

$\frac{-8}{12} \quad \frac{3}{12}$

$-\frac{2}{3} \quad \frac{1}{4}$

$(x + \frac{1}{2})(3x - 2)(4x + 1) = 0$

$x + \frac{1}{2} = 0$ $\quad 3x - 2 = 0$ $\quad 4x + 1 = 0$

$x = -\frac{1}{2}$ $\quad x = \frac{2}{3}$ $\quad x = -\frac{1}{4}$

Sep 28-10:04 AM

Practice Exercises pgs. 343-345:

3, 5, 7, 15, 19, 21, 27, 33, 37, 41, 43, 47, 49, 51, 53

15 total

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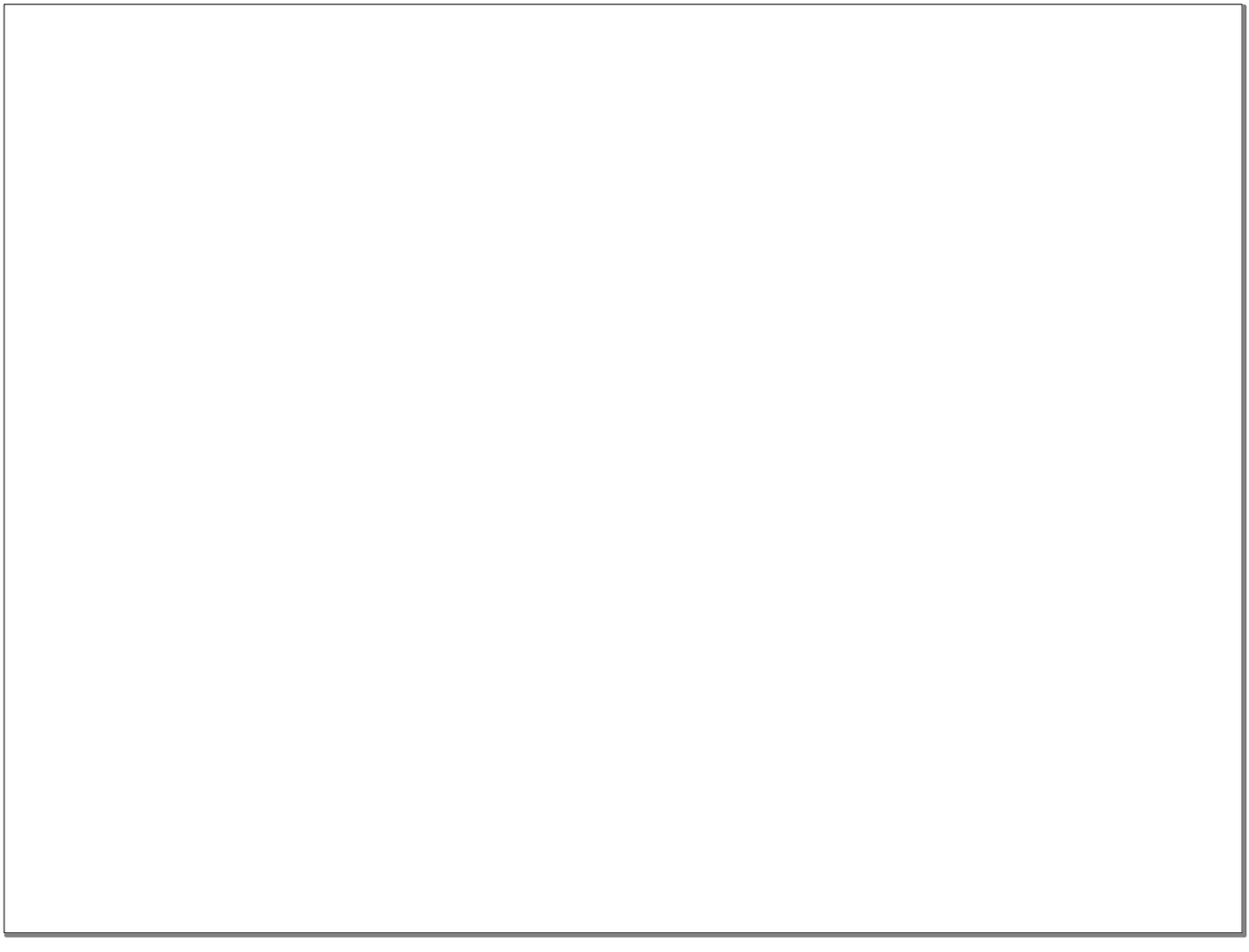

Sep 24-7:24 AM